ATME College of Engineering

Department of EEE
Emitting Elite Energy

Department of Electrical & Electronics Engineering

BEE302 – ELECTRIC CIRCUIT ANALYSIS

INTRODUCTION

- On the other hand, a voltage difference may occur across a component, depending the type of the component and the overall circuit.
- For unique representation of a node voltage, a reference node should be selected as a ground. However, the voltage across a component can always be defined uniquely since it is based on two or more (if the component has multiple terminals) points.
- In circuit analysis, voltages and currents are usually unknowns to be found. Since they are not known, in most cases, their direction can be arbitrarily selected.
- When the solution gives a negative value for a current or a voltage, it is understood that the initial assumption is incorrect. This is never a problem at all.
- For consistency, however, it is useful to follow a sign convention by fixing the voltage polarity and current direction for any given component. In the rest of this book, the current through a component is always selected to flow from the positive to the negative terminal of the voltage.

INTRODUCTION

5. Electric Energy and Power of a Component

Formally, we define the energy of the component as

$$w_d(t) = \int_0^t v_d(t')i_d(t')dt' \text{ (J)},$$

Power is defined as the product of its voltage and current.

$$p_d(t) = \frac{dw}{dt} = v_d(t)i_d(t) \text{ (W)}.$$

If p(t) > 0, the component absorbs energy at that specific time. Otherwise (i.e., if p(t) < 0), the component produces energy

Module 3a : RESONANT CIRCUITS

Objectives:

1. To study resonant circuits both in time and frequency domains.
2. Describe the conditions for electrical resonance.
3. Describe the mathematical strategy to develop the resonant frequency expression for a given resonant circuit.
4. Determine the resonant frequency of series, parallel, and series–parallel circuits.
5. Describe the quality factor.
6. Determine the quality factor of series, parallel, and series–parallel circuits.
7. Determine the three dB bandwidth from the resonant frequency and quality factor.
8. Decide whether a resonant circuit has a low Q or a high Q in order to select the 3 dB determination approach.

Introduction

AC Circuits made up of resistors, inductors and capacitors are said to be resonant circuits when the current drawn from the supply is in phase with the impressed sinusoidal voltage. Then

1. The resultant reactance or susceptance is zero. The circuit behaves as a resistive circuit.
2. The power factor is unity.

Applications in Communication circuits.
The ability of a radio or Television receiver

(1) To select a particular frequency or a narrow band of frequencies transmitted by broad casting stations.
(2) To suppress a band of frequencies from other broad casting stations, is based on resonance.

Resonance is desired in tuned circuits, design of filters, signal processing and control engineering. But it is to be avoided in other circuits. It is to be noted that if $R = 0$ in a series RLC circuit, the circuits acts as a short circuit at resonance and if $R = \infty$ in parallel RLC circuit, the circuit acts as an open circuit at resonance.

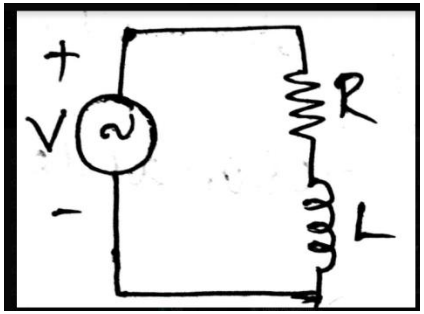
A second order series resonant circuit consists of R,L and C in series. At resonance, voltages across C and L are equal and opposite and these voltages are many times greater than the applied voltage. They may present a dangerous shock hazard.

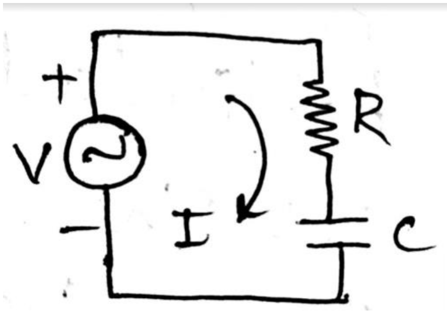
A second order parallel resonant circuit consists of R,L and C in parallel. At resonance, currents in L and C are circulating currents and they are considerably larger than the input current. Unless proper consideration is given to the magnitude of these currents, they may become very large enough to destroy the circuit elements.

Quality factor (Q-factor) / figure of merit

- Circuit efficiency is measured as Q - factor.
- Q – factor makes simple to compare various inductors & capacitors in terms of efficiency while designing such circuits.
- Definition of Q-factor,

$$Q = 2\pi \left(\frac{maximum\ energy\ stored\ in\ an\ energy\ storing\ element}{Energy\ dissipated\ per\ cycle\ in\ a\ resistor} \right)$$





Maximum energy stored in R-L circuit, $E_L = \frac{1}{2} L {I_m}^2$

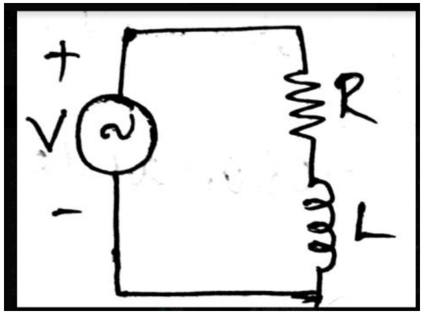
Energy dissipated/cycle = Power * time for one cycle

- Average power dissipated in resistor = $I^2 R = (\frac{I_m}{\sqrt{2}})^2 \ R$

- Energy dissipated/cycle = $(\frac{I_m}{\sqrt{2}})^2 \ R * T = \frac{1}{2} \ I_m^2 \ \frac{R}{f}$

- By definition, Q = $2\pi * \frac{\frac{1}{2} L I_m^2}{\frac{1}{2} I_m^2 \frac{R}{f}} = (2\pi f) \frac{L}{R} = \frac{\omega L}{R}$

- Therefore, Q = $\frac{\omega L}{R}$

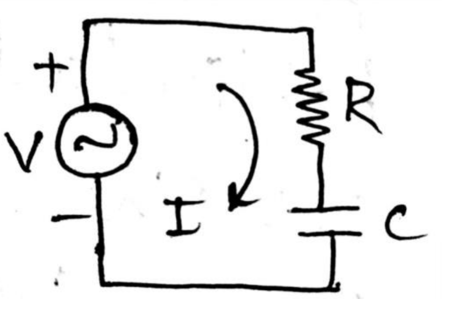


- Similarly consider a RC circuit
- Here, maximum energy stored, $E_C = \frac{1}{2}\, CV_m^2$
- By definition,

$$Q = 2\pi * \frac{\frac{1}{2}CV_m^2}{\frac{1}{2}\, I_m^2\, \frac{R}{f}} = (2\pi f)\frac{C}{R}\left(\frac{V_m}{I_m}\right)^2$$

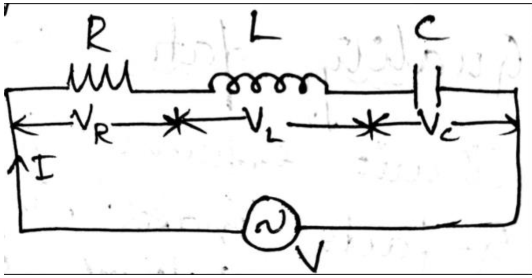
$$= \frac{\omega C}{R}\left(\frac{I_m^2 X_C^2}{I_m^2}\right) = \frac{\omega C}{R}\,\frac{1}{(2\pi f)^2 C^2} = \frac{\omega C}{R\omega^2 C^2}$$

- Therefore, $Q = \frac{1}{\omega RC}$



Series Resonance

i) Derivation of resonant frequency f_0



- Consider a series RLC circuit as shown in fig
- The impedance of series RLC circuit is $Z = \frac{V}{I}$
- Also, $Z = R + jX_L - jX_C = R + j(X_L - X_C) = R + j(\omega L - \frac{1}{\omega C})$

Where ω is frequency in radians/sec

- According to definition of resonance, at resonance, reactive part in impedance of series RLC circuit is zero. Let frequency be denoted by ω_0
- Therefore, At resonance, $\left[\omega_0 L - \frac{1}{\omega_0 C}\right] = 0$
- $\omega_0 L = \frac{1}{\omega_0 C}$ $\qquad\qquad$ $\omega_0^2 = \frac{1}{LC}$

- $\omega_0^2 = \frac{1}{LC}$
- Therefore, $\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec
- Since $\omega_0 = 2\pi f_0, we\ can\ write,$
- $f_0 = \frac{1}{2\pi\sqrt{LC}}$ HZ
- Thus, in series RLC circuit resonance may be produced either varying frequency for given constant values of L & C or varying either L & C or both for a given frequency
- At resonance inductive reactance is equal to the capacitive reactance.

Q-factor of series resonant circuit

The Q-factor of series resonant circuit is the Q-factor of inductor or capacitor in series resonant circuit at resonant frequency.

At resonance, $X_L = X_C$ hence the energy stored by both the elements would be same Q-factor of series resonant circuit is denoted by Q_0

At resonance, $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

Consider quality factor of series resonant circuit, we can write,

$Q_0 = \frac{\omega_0 L}{R}$, where $\boldsymbol{\omega_0 = \frac{1}{\sqrt{LC}}}$

$$Q_0 = \frac{\frac{1}{\sqrt{LC}} * L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Therefore, $\boldsymbol{Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}}$

Similarly, considering

$$Q_0 = \frac{1}{\omega_0 RC} = \frac{1}{\frac{1}{\sqrt{LC}} * RC}$$

$$= \frac{1}{R} * \frac{1}{\sqrt{\frac{C}{L}}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Therefore, $\boldsymbol{Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}}$ is the quality factor of series resonant circuit.

Thus, $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

Voltage across L & C at resonance

- At resonance $\boldsymbol{I_0 = \frac{V}{R}}$.
- The voltage across the inductance is,

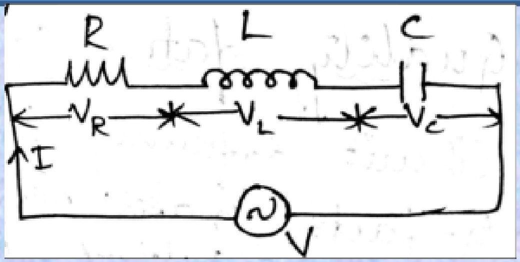
$$V_L = I_0(jX_L) = I_0(j\omega_0 L)$$
$$= j\left(\frac{V}{R}\right)\omega_0 L = j\left(\frac{\omega_0 L}{R}\right)V$$

Therefore $\boldsymbol{V_L = jQ_0 V}$

- Similarly voltage across the capacitance is,

$$V_C = I_0(-jX_C) = \frac{V}{R}\left(\frac{-j}{\omega_0 C}\right)$$
$$= -j\left(\frac{1}{\omega_0 RC}\right)V$$

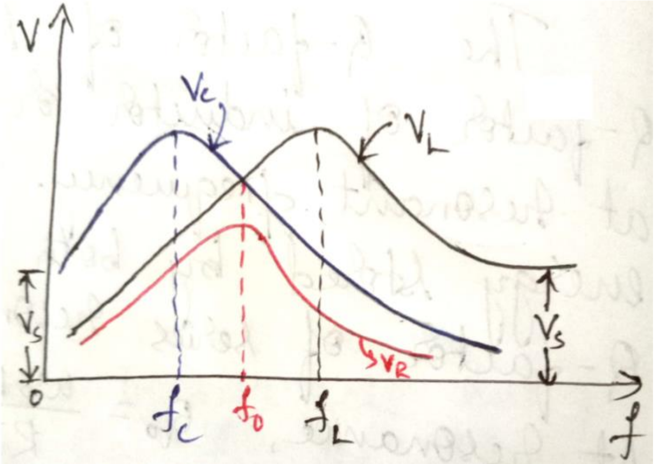
Therefore $\boldsymbol{V_c = -jQ_0 V}$

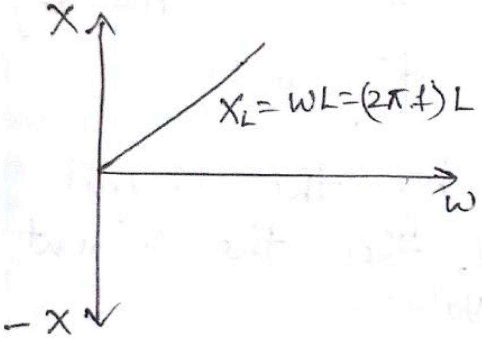


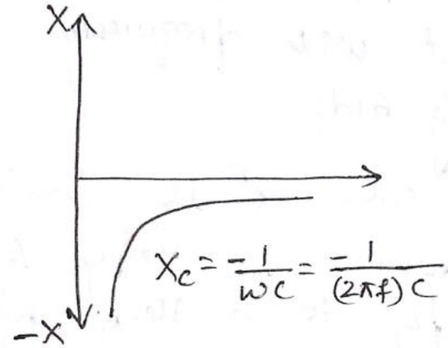
- Practically $\boldsymbol{Q_0 > 1}$.
- Voltage developed across L & C are more than applied voltage only at resonance.
- That means at resonance series RLC circuit acts as a voltage amplifier.
- Q_0 is referred as magnification factor.

Variation of voltages across L & C with frequency

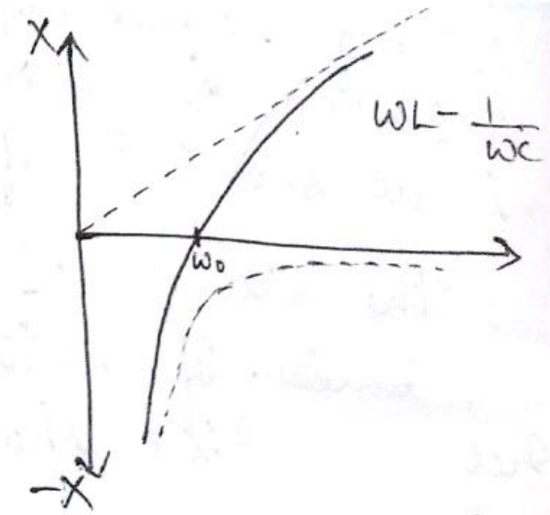
- Initially at f = 0, capacitor acts as open circuit & blocks current. Then across capacitor we have total input voltage say V_S.
- As **frequency increases**, $\boldsymbol{X_C}$ **decreases** & that of **X_L increases**. So that total $(X_C - X_L)$ **decreases** & current increases.
- As the current increases, voltage across R i.e., V_R increases & also both V_C & V_L increases.
- When frequency equals resonant frequency, the impedance equals R. Hence current approaches maximum value, so also the V_R reaches maximum value.

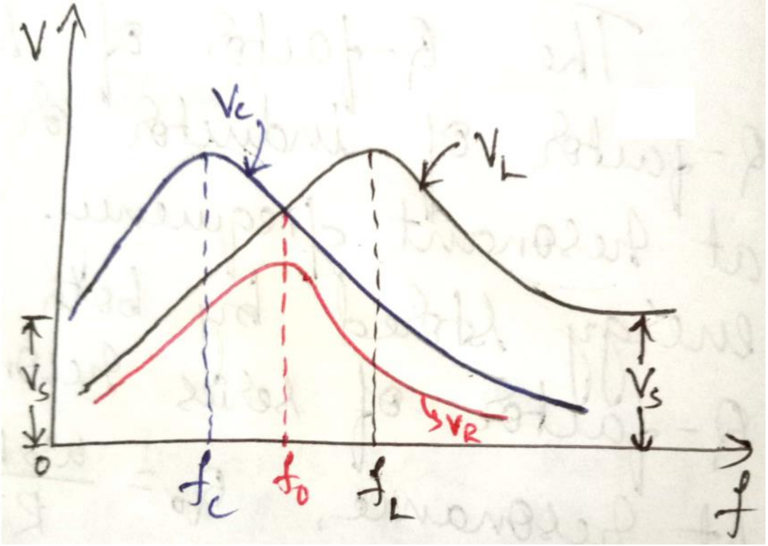






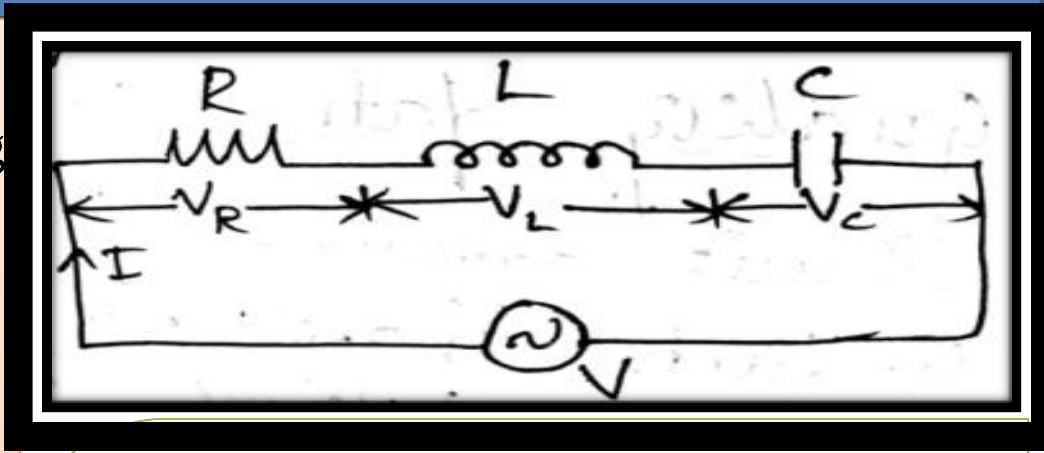
- As the **frequency is still increased above resonant frequency** X_L **further increases** & that of X_C **decreases**. This increases total reactance $(X_C - X_L)$, as a resultant ***impedance increases*** & the ***current decreases.***
- So V_R ***decreases*** & also V_C & V_L ***both decreases***.
- As frequency becomes very high, both V_R & V_C value approaches zero while V_L value approaches V_S.
- Variation of V_R, V_C & V_L with frequency is as shown in fig
- From fig, it is clear that, voltage across C & voltage across L is not maximum at resonant frequency.
- At **resonant frequency** f_0, the voltages V_C & V_L **are equal** in magnitude but opposite in phase.
- The voltage V_C **is maximum at** frequency f_C which is **less than** f_0 & the voltage V_L **is maximum at** frequency f_L which is **greater than** f_0.





Frequencies for maximum voltage across L & C

Consider a series RLC circuit as in fig



$$V = V_R + V_L + V_C$$

$$V_C = I(-jX_C) = \frac{I}{j\omega C}$$

$$V_L = I(jX_L) = I(j\omega L)$$

$$Z = R + j(X_L - X_C)$$

$$I = \frac{V}{Z} = \frac{V}{R + j(X_L - X_C)}$$

Therefore

$$V_C(j\omega C) = I = \frac{V}{R + j(X_L - X_C)}$$

$$= \frac{V_R + V_L + V_C}{R + j(\omega L - \frac{1}{\omega C})}$$

$$|V_C| = \frac{1}{\omega C}\left(\frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}\right)$$

The voltage across capacitor is as above. The frequency f_C at which V_C is maximum can be obtained by equating

$$\frac{dV_C^2}{d\omega} = 0$$

$$V_C^2 = \frac{1}{\omega^2 C^2} * \frac{V^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$= \frac{V^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

$$\frac{dV_C^2}{d\omega} = \frac{-V^2(2\omega R^2 C^2 + 2(\omega^2 LC - 1)(2\omega LC)}{[\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2]^2} = 0$$

$$2\omega R^2C^2 + 2(\omega^2LC - 1)(2\omega LC) = 0$$
$$2\omega[R^2C^2 + 2LC(\omega^2LC - 1)] = 0$$
$$R^2C^2 + 2\omega^2L^2C^2 - 2LC = 0$$
$$2\omega^2L^2C^2 = 2LC - R^2C^2$$
$$\omega^2 = \frac{2LC}{2L^2C^2} - \frac{R^2C^2}{2L^2C^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \text{ rad/sec}$$

Therefore, the frequency f_C at which inductor voltage V_C is maximum is given by,

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \quad \text{at } V_C \text{ maximum}$$

Similarly voltage across inductor is,

$$V_L = (j\omega L)\frac{V}{R + j(X_L - X_C)}$$

$$|V_L| = \frac{V\omega L}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2)}}$$

Squaring on both sides,

$$|V_L|^2 = \frac{V^2\omega^2L^2}{R^2 + (\omega L - \frac{1}{\omega C})^2)}$$

$$= \frac{V^2\omega^4L^2C^2}{\omega^2C^2R^2+(\omega^2LC-1)^2}$$

By differentiating V_L^2 $w.r.t$ ω & equating only numerator term to zero we have,

$$2\omega^2LC - \omega^2C^2R^2 - 2 = 0$$
$$\omega^2(2LC - C^2R^2) = 2$$
$$\omega^2 = \frac{2}{2LC - C^2R^2}$$
$$\omega^2 = \frac{1}{LC - \frac{C^2R^2}{2}}$$

$$\omega = \frac{1}{\sqrt{LC-\frac{C^2R^2}{2}}} \text{ rad / sec}$$

Therefore, the frequency f_L at which inductor voltage V_L is maximum is given by,

$$f_L = \frac{1}{2\pi}\frac{1}{\sqrt{LC-\frac{C^2R^2}{2}}}$$

$$V_L^2 = \frac{V^2\omega^4L^2C^2}{\omega^2R^2C^2 + (\omega^2LC-1)^2} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$$

$$\frac{dV_L^2}{d\omega} = \frac{[\omega^2R^2C^2+(\omega^2LC-1)^2]\,4V^2\omega^3L^2C^2 - V^2\omega^4L^2C^2\left[2\omega R^2C^2 + 2(\omega^2LC-1)\times 2\omega LC\right]}{[\omega^2R^2C^2+(\omega^2LC-1)^2]^2} = 0$$

$$[\omega^2R^2C^2+(\omega^2LC-1)^2]\,4\omega^3V^2L^2C^2 - 4V^2\omega^4L^2C^2\left[\tfrac{1}{2}\omega R^2C^2 + \omega LC(\omega^2LC-1)\right]$$

$$4V^2L^2C^2\omega^3\left\{\left[\omega^2R^2C^2+(\omega^2LC-1)^2\right] - \left[\frac{\omega^2R^2C^2}{2} + \omega^2LC(\omega^2LC-1)\right]\right\} = 0$$

$$\omega^2R^2C^2 + (\omega^2LC-1)^2 - \frac{\omega^2R^2C^2}{2} - \omega^2LC(\omega^2LC-1) = 0.$$

$$\frac{1}{2}\omega^2R^2C^2 + (\omega^2LC-1)\left[(\cancel{\omega^2LC}-1) - \cancel{\omega^2LC}\right] = 0.$$

$$\frac{1}{2}\omega^2R^2C^2 - (\omega^2LC-1) = 0.$$

$$\omega^2R^2C^2 - 2\omega^2LC + 2 = 0.$$

$$\omega^2\left[R^2C^2 - 2LC\right] = -2.$$

$$\omega^2 = \frac{-2}{R^2C^2 - 2LC}.$$

Divide by -2 in both numerator & denominator.

$$\omega^2 = \frac{1}{LC - \frac{R^2C^2}{2}}.$$

$$\therefore \boxed{\omega = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}} \text{ rad/sec} \; ; \quad \boxed{f = \frac{1}{2\pi}\cdot\frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}}$$

BANDWIDTH

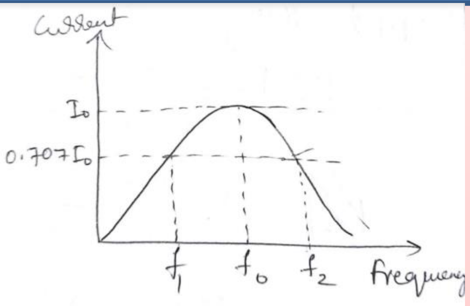
- It is defined as the width of resonant curve upto frequency at which the power in the circuit is half of its maximum value.
- From fig,
- $Bandwidth = (f_2 - f_1)$ Hz
- At resonance, $I_0 = \frac{V}{R}$.

& hence power in the circuit is also maximum,

- $P_0 = P_{max} = I_0^2 * R$
- Now, half of maximum power is given by,

$\sqrt{2}\cdot\sqrt{2}$

- $P^| = \frac{P_0}{2} = \frac{I_0^2 R}{2} = (\frac{I_0}{\sqrt{2}})^2 * R$
- So at the frequencies, where power in the circuit is half of its maximum value, current becomes $(\frac{1}{\sqrt{2}})$ times or 0.707 times of its maximum value.



- At resonant frequency, power in circuit is given by,
- $P_0 = P_{max} = I_0^2 * R$
- At frequency $f_{1,}$ power in circuit is half & it is given by,
- $P^| = \frac{I_0^2 R}{2}$ &
- Similarly, at frequency $f_{2,}$ power in circuit is half & it is given by,
- $P^| = \frac{I_0^2 R}{2}$ &
- Thus, $f_{1,}$ is called lower half-power frequency & $f_{2,}$ is called upper half-power frequency.

- The half-power frequencies are also referred as 3dB frequencies or 3dB points because the power at these frequencies is 3dB less than that at the resonance.
- The current in a series RLC circuit is given by equation,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}} \text{ ------(1)}$$

- At half power point, $I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,\frac{V}{R}$

because $I_0 = \frac{V}{R}$ at resonance

- Therefore, $\frac{1}{\sqrt{2}}\,\frac{V}{R} = \frac{V}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$

- $\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{2}\,R$
- Squaring on both sides,
- $R^2 + (\omega L - \frac{1}{\omega C})^2 = 2R^2$
- $(\omega L - \frac{1}{\omega C})^2 = R^2$
- $\left(\omega L - \frac{1}{\omega C}\right) = \pm R \text{ ------(2)}$
- Thus from above equation (2), at half-power frequencies f_1 & f_2, the reactive part of impedance of series RLC circuit is equal to resistive part of impedance.
- Equation (2) is quadratic in ω, hence we can write,
- $\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = +R \text{ ------(3)}$
- $\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R \text{ ------(4)}$

- $\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = +R ----- -(3)$
- $\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R ----- -(4)$
- Adding equations (3) & (4), we have
- $(\omega_1 + \omega_2)L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$
- Therefore, $(\omega_1 + \omega_2)L$ - $(\frac{\omega_1 + \omega_2}{\omega_1\omega_2})\frac{1}{C} = 0$
- $(\omega_1 + \omega_2)L = (\frac{\omega_1 + \omega_2}{\omega_1\omega_2})\frac{1}{C}$
- $\omega_1\omega_2 = \frac{1}{LC} ----- -(5)$
- But from condition of resonance,

$\omega_0 = \frac{1}{\sqrt{LC}}$

- Therefore, $\omega_1\omega_2 = \omega_0^2$
- i.e., $f_1 f_2 = f_0^2$
- This shows that the resonant frequency is the geometric mean of two half power frequencies.
- Thus, $f_0 = \sqrt{f_1 f_2}$
- **Subtracting equation (3) & (4), we get**
- $(\omega_2 - \omega_1)L + \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right)\frac{1}{C} = 2R$
- $(\omega_2 - \omega_1) + (\frac{\omega_2 - \omega_1}{\omega_1\omega_2})\frac{1}{LC} = \frac{2R}{L}$
- **From equation (5),** $\omega_1\omega_2 = \frac{1}{LC}$
- $(\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L}$
- $(\omega_2 - \omega_1) = \frac{R}{L}$
- **Bandwidth,** $(f_2 - f_1) = \frac{R}{2\pi L}$

Selectivity

- Selectivity of a resonant circuit is defined as the ability of a circuit to discriminate or distinguish between desired and undesired frequencies.
- Selectivity is also defined as the ratio of resonant frequency to the bandwidth of resonant circuit.
- Therefore,

Selectivity = $\frac{Resonant\ frequency}{Bandwidth} = \frac{f_0}{(f_2 - f_1)}$

- Also, Bandwidth, $(f_2 - f_1) = \frac{R}{2\pi L}$
- Therefore,

Selectivity = $\frac{f_0}{(f_2 - f_1)} = \frac{f_0}{(\frac{R}{2\pi L})} = \frac{(2\pi f_0)L}{R} = \frac{\omega_0 L}{R}$

- Where Q-factor, $Q_0 = \frac{\omega_0 L}{R}$
- Therefore, Selectivity = $\frac{\omega_0 L}{R} = Q_0$
- Thus, selectivity of series resonant circuit is directly proportional to the quality factor of circuit at resonant frequency.
- Selectivity = $Q_0 = \frac{f_0}{B.W} = \frac{f_0}{(f_2 - f_1)}$
- Therefore,

Bandwidth = $(f_2 - f_1) = \frac{f_0}{Q_0}$

Parallel Resonance / Anti-Resonance

- A parallel circuit is said to be in resonance when **applied voltage & resulting current in phase** that gives **unity power factor condition.**

a) Derivation of resonant frequency (f_{ar}) for the circuit R-L parallel with C.

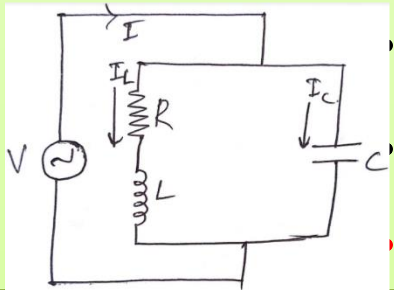
- Consider parallel resonant circuit as shown in fig with applied voltage V & total resulting current I.
- The admittance of branch containing L & R_L is given by,
- $Y_L = \frac{1}{R_L + jX_L}$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} = \frac{R_L - jX_L}{R_L^2 + \omega^2 L^2} ---- -(1)$$

- The admittance of branch containing C is given by

$$Y_C = \frac{1}{-jX_C} = j\frac{1}{X_C}$$

$$= j\frac{1}{\left(\frac{1}{\omega C}\right)} = j\omega C ---- -(2)$$



- Hence, total admittance of parallel circuit is given by,

$$Y = Y_L + Y_C$$

- Therefore, $Y = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + j\omega C$

$$Y = \frac{R_L}{R_L^2 + \omega^2 L^2} + j\left[\omega C - \frac{\omega L}{R_L^2 + \omega^2 L^2}\right] - (3)$$

- At resonance, imaginary part i.e, susceptance becomes zero. Let the resonant frequency of parallel resonant circuit be denoted by ω_{ar}.

Thus at $\omega = \omega_{ar}$,

- $\omega_{ar} C = \frac{\omega_{ar} L}{R_L^2 + \omega_{ar}{}^2 L^2} = 0$
- $\omega_{ar} C = \frac{\omega_{ar} L}{R_L^2 + \omega_{ar}{}^2 L^2}$
- $C(R_L^2 + \omega_{ar}{}^2 L^2) = L$
- $(R_L^2 + \omega_{ar}{}^2 L^2) = \frac{L}{C} ------ -(4)$

$$\omega_{ar}{}^2 L^2 = \frac{L}{C} - R_L^2$$

$$\omega_{ar}{}^2 = \frac{1}{LC} - \frac{R_L^2}{L^2}$$

$$\omega_{ar} = \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

$$f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} ------ -(5)$$

f_{ar} is the resonant frequency. This parallel resonance is possible as long as $\frac{1}{LC} > \frac{R_L^2}{L^2}$, Otherwise f_{ar} will be imaginary.

$Y = \frac{1}{Z} = \frac{R_L}{R_L^2+\omega^2 L^2}$ Therefore, $Z = \frac{R_L^2+\omega^2 L^2}{R_L}$

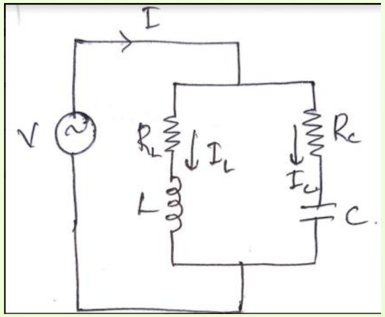
But we have, $\boldsymbol{R_L^2 + \omega^2 L^2 = \frac{L}{C}}$

Therefore, $\boldsymbol{Z_{ar}} = \boldsymbol{\frac{L}{CR_L}}$ $------ -(6)$

Consider equation(4), $R_L^2+\omega_{ar}{}^2 L^2 = \frac{L}{C}$

$R_L^2(1 + \boldsymbol{\frac{\omega_{ar}{}^2 L^2}{R_L^2}}) = \frac{L}{C}$ $R_L(1 + \boldsymbol{Q_0^2}) = \boldsymbol{\frac{L}{CR_L}} = \boldsymbol{Z_{ar}}$

b) Derivation of resonant frequency (f_{ar}) for the circuit R-L parallel with R-C.



- Consider a general parallel RLC circuit shown in fig with applied voltage V & total resulting current I.
- The condition of parallel resonance is that the impedance of the parallel combination is purely resistive.
- The admittance of branch containing L & R_L is given by
- $Y_L = \frac{1}{R_L+jX_L} = \frac{R_L-jX_L}{R_L^2+X_L^2}$ … by rationalizing
- The admittance of branch containing C & R_C is given by
- $Y_C = \frac{1}{R_C-jX_C} = \frac{R_C+jX_C}{R_C^2+X_C^2}$ … by rationalizing
- Therefore, Total admittance Y is given by,
- $Y = Y_L + Y_C$
- $Y = \frac{R_L-jX_L}{R_L^2+X_L^2} + \frac{R_C+jX_C}{R_C^2+X_C^2}$
- $Y = \left(\frac{R_L}{R_L^2+X_L^2} + \frac{R_C}{R_C^2+X_C^2}\right) + j\left(\frac{X_C}{R_C^2+X_C^2} - \frac{X_L}{R_L^2+X_L^2}\right)$
- At resonance, susceptance(imaginary part of admittance) becomes zero. Therefore, we have condition as,
- $\left(\frac{X_C}{R_C^2+X_C^2} - \frac{X_L}{R_L^2+X_L^2}\right) = 0$
- $\frac{X_C}{R_C^2+X_C^2} = \frac{X_L}{R_L^2+X_L^2}$

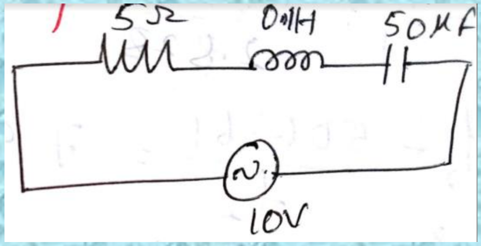
- $\dfrac{\frac{1}{\omega_{ar}C}}{R_C^2+(\frac{1}{\omega_{ar}C})^2} = \dfrac{\omega_{ar}L}{R_L^2+\omega_{ar}{}^2L^2}$
- $R_L^2 + \omega_{ar}{}^2L^2 = (\omega_{ar}C)\,(\omega_{ar}L)\,[R_C^2 + \left(\frac{1}{\omega_{ar}{}^2C^2}\right)]$
- $R_L^2 + \omega_{ar}{}^2L^2 = \omega_{ar}{}^2LC\,R_C^2 + \frac{L}{C}$
- $\omega_{ar}{}^2L^2 - \omega_{ar}{}^2LC\,R_C^2 = \frac{L}{C} - R_L^2$
- $\omega_{ar}{}^2LC\left(\frac{L}{C} - R_C^2\right) = \left(\frac{L}{C} - R_L^2\right)$
- $\omega_{ar}{}^2 = \frac{1}{LC}\left[\frac{\left(\frac{L}{C} - R_L^2\right)}{\left(\frac{L}{C} - R_C^2\right)}\right]$
- $\omega_{ar} = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}$
- $f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}}$
- If $R_L = R_C = \sqrt{\frac{L}{C}}$
- $f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{(\frac{L}{C})^2-\frac{L}{C}}{(\frac{L}{C})^2-\frac{L}{C}}}$
- $f_{ar} = \frac{1}{2\pi\sqrt{LC}}$
- Where f_{ar} is frequency of resonance. The values of R_L & R_C are in general very small.

List of the formulae of Series Resonant Circuit

- $f_0 = \frac{1}{2\pi\sqrt{LC}}$
- At resonance, $I_0 = \frac{V}{R}$
- $Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$; B.W $= (f_2 - f_1) = \frac{R}{2\pi L} = \frac{f_0}{Q_0}$
- $V_L = I_0 X_L$; $V_C = I_0 X_C$
- B.W $= 2\Delta f$; $f_1 = f_0 - \Delta f$; $f_2 = f_0 + \Delta f$
- $f_0 = \sqrt{f_1 f_2}$
- $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
- $f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$ $f_L = \frac{1}{2\pi}\frac{1}{\sqrt{LC - \frac{C^2R^2}{2}}}$

Numerical on Series Resonant Circuit

1. A coil of 5Ω resistance & 0.1H inductance is connected in series with a capacitance of 50μF across an AC supply of 10V of variable frequency. Determine i) Resonant frequency ii) Current at resonance iii) Q-factor of the coil. iv) Bandwidth v) Voltage across L & C vi) Compute the lower & upper frequency limits.



Solution:

i) The series resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 * 50\mu}} = \mathbf{71.176\ Hz}$$

ii) Current at resonance, $I_0 = \frac{V}{R} = \frac{10}{5} = \mathbf{2A}$

iii) $Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi * 71.176 * 0.1}{5} = \mathbf{8.944 = 9}$

iv) B.W = $\frac{f_0}{Q_0} = \frac{71.176}{9} = \mathbf{7.91Hz}$

v) $V_L = I_0 X_L = 2 * (2\pi * 71.176 * 0.1) = \mathbf{89.44V}$

$$V_C = I_0 X_C = 2 * \frac{1}{2\pi * 71.176 * 50\mu} = \mathbf{89.44V}$$

vi) The lower frequency is given by $\boldsymbol{f_1 = f_0 - \Delta f}$

where **B.W = 2**$\boldsymbol{\Delta f}$

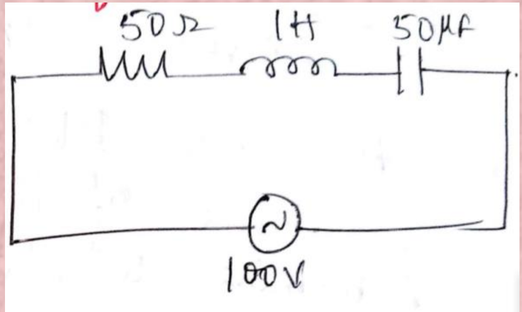
Therefore, $\Delta f = \frac{B.W}{2} = \frac{7.91}{2} = \mathbf{3.96\ Hz}$

Therefore, $f_1 = f_0 - \Delta f$ = 71.176 – 3.96 = $\mathbf{67.216\ Hz}$

The upper frequency is given by $f_2 = f_0 + \Delta f$

$$f_2 = 71.176 + 3.96 = \mathbf{75.136\ Hz}$$

2. A series RLC circuit has R=50Ω, L = 1H, C=50μF connected across ac variable frequency of 100V. Calculate resonant frequency & half power frequencies.



Solution:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 * 50\mu}} = \mathbf{22.51\ Hz}$$

w.k.t $f_0 = \sqrt{f_1 f_2}$ $\qquad f_0^2 = f_1 f_2$ $\qquad f_1 = \frac{506.61}{f_2}$

w.k.t, B.W = $(f_2 - f_1) = \frac{f_0}{Q_0}$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi * 22.51 * 1}{50} = \mathbf{2.83 = 3}$$

$$(f_2 - f_1) = \frac{22.51}{3} = \mathbf{7.503\ Hz}$$

Therefore, $f_2 - \frac{506.61}{f_2} = 7.503$

$f_2^2 - 7.503 f_2 - 506.61 = 0$; $\qquad$ Therefore, $f_2 = \mathbf{26.57\ Hz}$

Therefore, $f_1 = \frac{506.61}{f_2} = \frac{506.61}{26.57} = \mathbf{19.1\ Hz}$

4. The voltage applied to the series RLC circuit is 0.85V. The Q of the coil is 50 & the value of capacitor is 320PF. The resonant frequency of the circuit is 175Khz. Find the value of inductance, the circuit current & voltage across capacitor & inductor.

Solution:

V = 0.85V; Q = 50; C = 320Pf; $f_0 = 175\ kHz$

$$Q_0 = \frac{1}{\omega_0 RC} \qquad R = \frac{1}{2\pi * 175 * 10^{-3} * 320 * 10^{-12} * 50} = \mathbf{56.84}\Omega$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \sqrt{\frac{L}{C}} = 50 * 56.84$$

Therefore, L = 2.58mH

$$I_0 = \frac{V}{R} = \frac{0.85}{56.84} = \mathbf{14.95\ mA}$$

$$V_L = I_0 \omega_0 L = 14.95 * 10^{-3} * 2\pi * 175 * 10^3 * 2.58 * 10^{-3} = \mathbf{42.74\ V}$$

$$V_C = I_0/\omega_0 C = 14.95 * 10^{-3} * 2\pi * 175 * 10^3 * 320 * 10^{-12} = \mathbf{42.488\ V}$$

$$f_C = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{2.58*10^{-3}*320*10^{-12}} - \frac{56.84^2}{2(2.58*10^{-3})^2}} = \mathbf{175.14\ kHz}$$

$$f_L = \frac{1}{2\pi}\frac{1}{\sqrt{LC - \frac{C^2R^2}{2}}} = \frac{1}{2\pi}\frac{1}{\sqrt{2.58*10^{-3}*320*10^{-12} - \frac{(320*10^{-12})^2(56.84)^2}{2}}} = \mathbf{175.17\ kHz}$$

3. A series RLC circuit includes 1μF capacitor & a resistance of 16Ω. If the bandwidth is 500 rad/sec. Determine ω_r, Q & L.

Solution:

B.W = 500 rad/sec = $\omega_2 - \omega_1$

R = 16Ω

C = 1μF

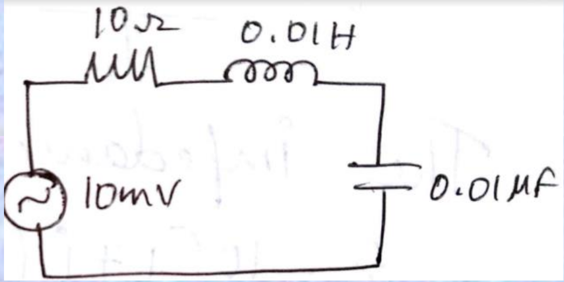
B.W = $\omega_2 - \omega_1 = \frac{R}{L}$

L = $\frac{16}{500} = 32\ mH$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{32 * 10^{-3} * 1 * 10^{-6}}} = \mathbf{5590.17\ rad\ /sec}$$

$$Q = \frac{\omega_r}{B.W} = \frac{5590.17}{500} = \mathbf{11.1803}$$

5. A series RLC circuit has R = 10Ω, L = 0.01H & C=0.01µF & it is connected across 10mV supply. Calculate i) $f_{0,}$ ii) Q_0 iii) Bandwidth iv) f_1 & f_2 v) I_0



Solution:

i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01*0.01\mu}} = \mathbf{15.92\ kHz}$

$ii)\ \ Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi*15.92*10^3*0.01}{10} = \mathbf{100}$

iii) B.W = $(f_2 - f_1) = \frac{f_0}{Q_0} = \frac{15.92*10^3}{100} = \mathbf{159.2\ Hz}$

iv) B.W = $2\Delta f$

$\Delta f = \frac{159.2}{2} =$ **79.6 Hz**

$f_1 = f_0 - \Delta f = 15.92 * 10^3 - 79.6 = \mathbf{15.84\ kHz}$

$f_2 = f_0 + \Delta f = 15.92 * 10^3 + 79.6 = \mathbf{15.999\ kHz}$

v) $I_0 = \frac{V}{R} = \frac{10*10^{-3}}{10} = \mathbf{1mA}$

List of the formulae of Parallel Resonant Circuit

- $Q_{ar} = \frac{\omega_{ar} L}{R} = \frac{1}{\omega_{ar} CR}$; Bandwidth = $f_2 - f_1 = \frac{f_{ar}}{Q_0}$
- $Z_0 = \frac{L}{RC}$ or $Z_{ar} = \frac{L}{R_L C}$
- $f_{ar} = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$
- $f_{ar} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$; $f_{ar} = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{1}{Q_0^2}}$
- The current at parallel resonance, $I_0 = \frac{V}{Z_{ar}}$
- $Z_{ar} = R_L (1 + Q_0^2)$

Numerical on Parallel Resonant Circuit

6. In a parallel resonant circuit R,L,C half power frequencies are 103 & 118 rad/sec respectively. The magnitude of impedance at 105 rad/sec is 10Ω. Find R,L & C

Solution:-

$$\omega_1 = 103\, rad/sec$$

$$\omega_2 = 118\, rad/sec$$

$$\omega_{ar} = 105\, rad/sec$$

$$Z_{ar} = 10\ \Omega$$

BW = $\omega_2 - \omega_1$

$= 118 - 103 = 15\, rad/sec$

$$Q_{ar} = \frac{\omega_{ar} L}{R}$$

$$Q_{ar} = \frac{\omega_{ar}}{B.W} = \frac{105}{15} = 7$$

Therefore, $\frac{L}{R} = \frac{7}{105}$

Therefore, L = $\frac{7R}{105}$

$$Z_0 = \frac{L}{RC} = \frac{7}{105} * \frac{R}{RC} = \frac{7}{105C}$$

$10 = \frac{7}{105\, C}$

Therefore, C = 6.67 mF

$$Q_{ar} = \frac{1}{\omega_{ar} CR}$$

$$R = \frac{1}{7 * 105 * 6.67 * 10^{-3}}$$

$$= 0.204\Omega$$

L = $\frac{7*0.204}{105} = 13.6\, mH$

7. A two branch anti-resonant circuit contains L = 0.4H & C = 40µF. Resonance is to be achieved by variation of R_L & R_C. Calculate the resonance frequency for the following cases.

i) $R_L = 120\Omega, R_C = 80\Omega$

ii) $R_L = 80\Omega, R_C = 0\Omega$

iii) $R_L = R_C = 100\Omega$

Solution:

The resonant circuit is shown in fig,

i) L = 0.4H, C = 40µF, $R_L = 120\Omega, R_C = 80\Omega$

$$f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$f_{ar} = \frac{1}{2\pi\sqrt{0.4*40*10^{-6}}}\sqrt{\frac{120^2-(\frac{0.4}{40*10^{-6}})}{80^2-(\frac{0.4}{40*10^{-6}})}}$ With $R_C = 80\Omega,$ the frequency comes out to be imaginary which is absurd

ii) L = 0.4H , C = 40µF, $R_L = 80\Omega, R_C = 0.$

$$f_{ar} = 23.87\ Hz$$

iii) $R_L = \ R_C = 100\Omega$

$$f_{ar} = 39.7887\ Hz$$

8. A coil of inductance 10H & 10Ω resistance is connected in parallel with 100 Pf capacitor. The combination is applied with a voltage of 100V. Find resonant frequency & current at resonance

Solution: $R_L = 10\Omega, L = 10H, C = 100PF$

$$f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{10*100*10^{-12}} - \frac{10^2}{10^2}} = 5.033 \text{ kHz}$$

At resonant frequency, the impedance of parallel resonant circuit is

$$Z_{ar} = \frac{L}{R_L C} = \frac{10}{10*100*10^{-12}} = 10 * 10^9\ \Omega$$

The current at parallel resonance, $I_0 = \frac{V}{Z_{ar}} = \frac{100}{10*10^9} = 10nA$

9. A parallel circuit has a fixed capacitor & variable inductor having constant quality factor of 4. Find value of inductance & capacitance for circuit impedance of 1000Ω at resonating frequency 2.4 Mhz. What is bandwidth of circuit?

Solution: $Q_0 = 4$; $Z_{ar} = 1000\Omega$;
$f_{ar} = 2.4$ Mhz

$$Z_{ar} = \frac{L}{R_L C}$$

Also, $Z_{ar} = R_L\,(1 + Q_0^2)$
$1000 = R_L\,(1 + 4^2)$
Therefore, $R_L = 58.82\ \Omega$

$Z_{ar} = \frac{L}{R_L C}$ =1000

$$\frac{L}{C} = 1000\ R_L$$

$$\frac{L}{C} = 1000 * 58.82$$

Therefore, $\frac{L}{C} = 58.82 * 10^3$ ---------(1)

$$f_{ar} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

Also $f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{Q_0^2}}$ = $f_{ar} = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{16}}$

$$(2.4 * 10^6)^2 = \frac{1}{4\pi^2(LC)}\left(\frac{15}{16}\right)$$

Therefore, LC = $4.1227 * 10^{-15}$
----------(2)
From equation (1) & (2),
L = 15.57 µH;
C = 0.264nF

Bandwidth = $f_2 - f_1 = \frac{f_{ar}}{Q_0}$

= $\frac{2.4 * 10^6}{4}$ = 0.6 Mhz

10 Two impedances $Z_1 = 20 + j10$ & $Z_2 = 10 - j30$ are connected in parallel & this combination is connected in series with $Z_3 = 30 + jX$. Find the value of X which will produce resonance.

Solution: The total impedance is given by,

$$Z = Z_3 + (Z_1||Z_2)$$

$$Z = 30 + jX + \frac{(20 + j10) * (10 - j30\ )}{(20 + j10) + (10 - j30\ )}$$

$$Z = \left[30 + \frac{250}{13}\right] + j\ [X - \frac{50}{13}]$$

The Circuit shown in fig above will resonate, if imaginary part is zero,

Therefore, X – $\frac{50}{13} = 0$

Therefore, X = $\frac{50}{13} = 3.646\Omega$

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18EE32 – ELECTRIC CIRCUIT ANALYSIS

Module 3b: Transient Analysis

Introduction

1. A network in which branch currents & node voltages are not changing with respect to time is said to be in steady state.

2. When a network is switched from one condition to another by change in applied voltage or by change in applied voltage or by change in one of the circuit elements, during a period of time, branch currents & voltages change from their former value to new one. This time interval is called transition period. The response or the output of network during transition period is called transient response of network.

3. If after transition period, network condition is not disturbed, then the network attains steady state at infinite time.

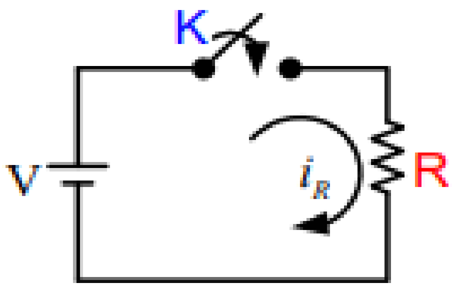
4. Energy storing elements such as inductor & capacitor results in a differential equation whose solution consists of 2 parts, the complementary function & Particular solution.

5. The Complementary function represents transient part of solution which decays with time, while remaining term represents steady state part of solution.

Initial Conditions in elements

Resistor

Consider a circuit which consists of resistor R connected as shown in Figure 1.1. The circuit resistor R is connected by a voltage source V in series with switch K as shown in Figure.

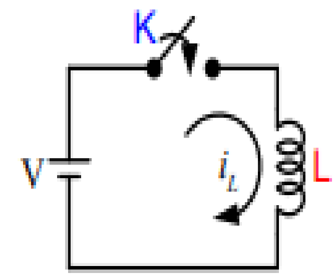


Series resonance circuit

When the switch K is closed at t=0 the current I is flowing in a circuit and is given by I = V/R

Inductor

Consider a circuit which consists of inductor L connected as shown in Figure 1.2. The inductor L is connected by a voltage source V in series with switch K as shown in Figure. When the switch K is closed at t=0 the current flowing in a inductor at t = 0+ is zero the inductor acts as a open circuit at t = 0+ which is as shown in Figure 1.3.

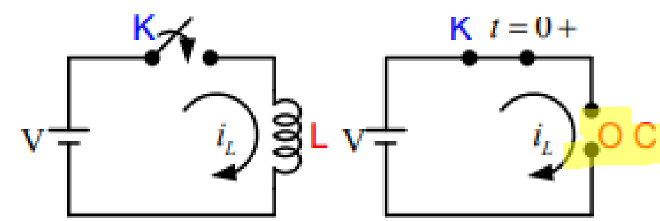


Inductor circuit

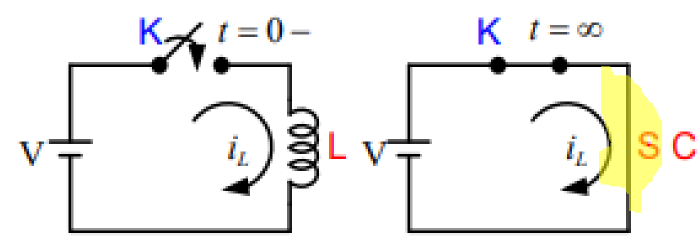
The final-condition equivalent circuit of an inductor is derived from the basic relationship

$$v = L\frac{di}{dt}$$

Under steady state condition, rate of change of current flowing in inductor is di/ dt = 0. This means, v = 0 and hence L acts as short at t = ∞. The equivalent circuits of an inductor at t = ∞ is as shown in Figure 1.4



Inductor circuit at $t = 0+$



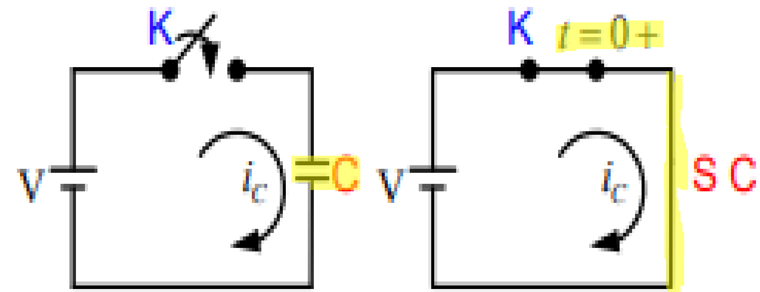
Inductor circuit at $t = 0+$

Capacitor

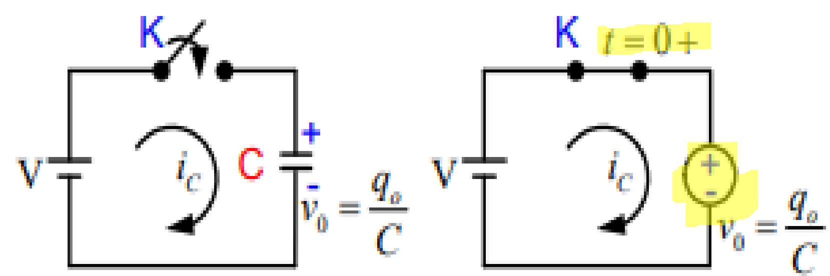
Consider a circuit which consists of capacitor C connected as shown in Figure 1.5. The capacitor is connected by a voltage source V in series with switch K as shown in Figure 1.5. When the switch K is closed at t=0 capacitor C acts as short circuit and current flows in a capacitor instantaneously.

$q = cV$

If the capacitor is initially charged with charge q0 coulombs at t=0-, then at t=0+ the capacitor is equivalent to voltage source v0 = q0/ c which is as shown in Figure 1.6



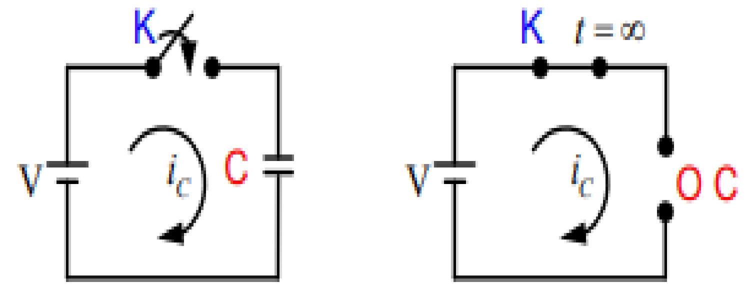
Capacitor circuit at t = 0+



Capacitor circuit at t = 0+

The final condition of capacitor circuit is derived from the following relationship. The voltage across capacitor is $v = C\frac{dv}{dt}$

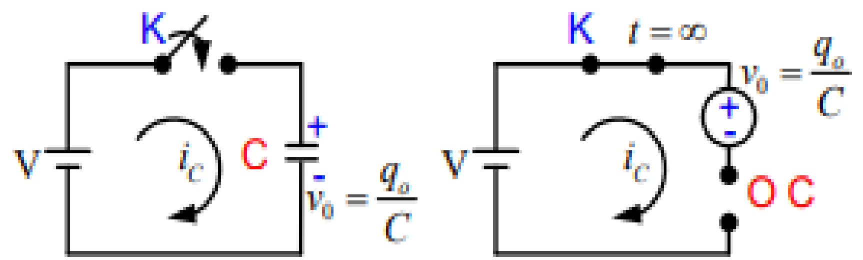
Under steady state condition, rate of change of voltage capacitor is dv / dt = 0. This means, v = 0 and hence C acts as open circuit at t = ∞. The equivalent circuits of a capacitor at t = ∞ is as shown in Figure 1.7



Capacitor circuit at t = ∞

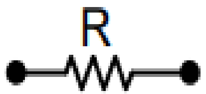
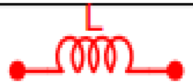
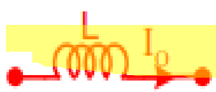
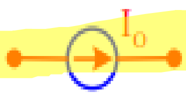
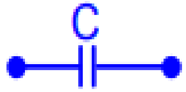
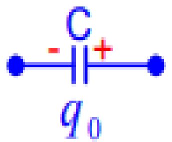
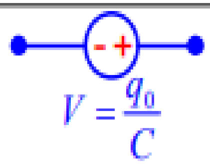
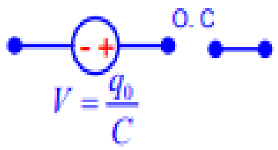
If the capacitor is initially charged with voltage v0 then the final condition at t = ∞ of a capacitor circuit is replaced with voltage source v0 with open circuit which is as shown in Figure 1.8

$$v = C\frac{dv}{dt}$$



Capacitor circuit at t = ∞

Table 1.1: Initial and Final Conditions

at t=0-	at t=0+	at $t = \infty$
		
		
		
		
		

Procedure for Evaluating Initial Conditions:

1. Before closing or opening the switch at t=0- find the history of the network, at t=0- find i(0-), v(0-), i.e., current through inductor and voltage across the capacitor before switching

2. Draw the circuit after switching operation at t=0+.

3. Replace inductor with open circuit or by current source having source

4. Replace capacitor with short circuit or with a voltage source $V_C = \frac{q_0}{C}$ if it has an initial charge q_0.

5. Find i(0+), and v(0+) at t=0+

6. Obtain an expression for di/dt and find di/dt at t=0+

7. Obtain an expression for $\frac{d^2i}{dt^2}$ and find $\frac{d^2i}{dt^2}$ at t=0+

8. Similarly determine voltages across circuit elements and its derivatives.

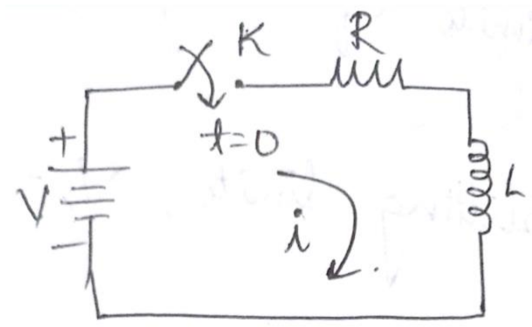
DC Excitation to series R-L Circuit

i) D.C response of R-L series circuit

- At t=0-, $i_L(0^-) = 0$

Since the current through inductor cannot change instantaneously, $i_L(0^+) = 0$

- Let initial current be I_0. Here $I_0 = 0$.
- Assume switch K is closed at t=0
- After closing the switch, apply KVL



$$V = iR + L\frac{di}{dt}$$

$$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}$$

$$\left(\frac{V}{R} - i\right)dt = \frac{L}{R}\,di$$

$$\left(\frac{R}{L}\right)dt = \frac{di}{(\frac{V}{R} - i)}$$

Integrating both sides

$$\int \frac{R}{L}\,dt = \int \frac{di}{(\frac{V}{R} - i)}$$

$\frac{R}{L}\,t$ = -ln[$\frac{V}{R} - i$] + $K^{|}$ -------(1)

To find $K^{|}$, At t = 0, i = I_0 = 0.

0 = -ln ($\frac{V}{R}$) + $K^{|}$

Therefore, $K^{|}$ = ln ($\frac{V}{R}$) --------(2)

Substitute equation (2) in (1), we get

$\frac{R}{L}$t = -ln [$\frac{V}{R}$-i] + ln [$\frac{V}{R}$]

$$\frac{R}{L}\ t = \ln[\frac{\frac{V}{R}}{(\frac{V}{R} - i)}]$$

Taking antilog, $e^{\left(\frac{R}{L}\right)t} = \frac{V/R}{(\frac{V}{R} - i)}$

$\frac{V}{R} - i = \frac{V}{R}\ .\ e^{-\left(\frac{R}{L}\right)t}$

$i = \frac{V}{R} - \frac{V}{R}\ .\ e^{-\left(\frac{R}{L}\right)t}$

$\frac{V}{R}$ is steady state part

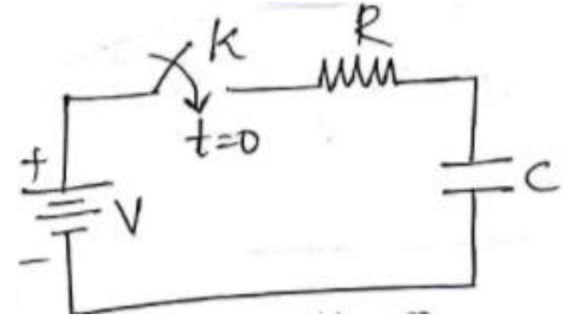
$i = \frac{V}{R}(1 - e^{-\left(\frac{R}{L}\right)t})$

$\frac{V}{R}\ .\ e^{-\left(\frac{R}{L}\right)t}$ is transient part.

DC Excitation to series R-C Circuit

i) D.C response of R-C series circuit

→ Initially switch K is in open state so no charge on condenser.



→ At t=0, K is closed, capacitor acts as short circuit, so current at the time of switching is high. ∴ $V_c = 0$.

→ At $t=0^+$; Voltage across capacitor = 0.
i is maximum at $t=0^+$.

→ As the capacitor starts charging, the voltage across capacitor V_c starts increasing & charging current starts decreasing. After some time, when the capacitor charges to V volts, it achieves steady state. In steady state, it acts as a open circuit so current will be zero finally.

→ After switching instant, applying KVL, $V = V_R + V_C$

$$V = i \cdot R + V_C$$

But current i can be written as $i = C\frac{dV_C}{dt}$

$$V = \left(C\frac{dV_C}{dt}\right)R + V_C$$

$$V - V_C = RC\frac{dV_C}{dt}.$$

$$(V - V_C)\,dt = (RC)\,dV_C$$

$$\int \frac{dt}{RC} = \int \frac{dV_C}{(V-V_C)}$$

$$\frac{1}{RC}t = -\ln(V - V_C) + k' \longrightarrow (1).$$

To find k', At. $t = 0$; $V_C = 0$.

$$0 = -\ln V + k'.$$

$$\therefore k' = \ln[V].$$

Substitute k' in eqn (1).

$$\frac{1}{RC}t = -\ln(V - V_C) + \ln[V].$$

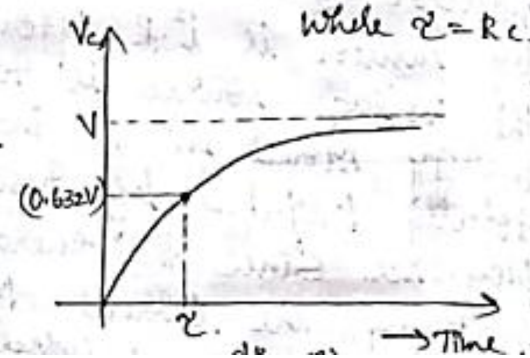
$$\frac{t}{RC} = \ln\left[\frac{V}{V - V_C}\right]$$

Taking antilog, $e^{(1/RC)t} = \frac{V}{V - V_C}$

$$V - V_C = Ve^{-t/RC}$$

$$\therefore \underline{V_C = V(1 - e^{-t/RC})}.$$

where $\tau = RC$.



When the steady state is achieved,
total charge on the capacitor is Q coulombs.

$$\therefore V = \frac{Q}{C}$$

$|||^{ly}$ at any instant, $V_c = \frac{q}{C}$, where q is instantaneous charge

$$\frac{q}{C} = \frac{Q}{C}\left[1 - e^{-t/RC}\right]$$

$$\therefore q = Q\left[1 - e^{-t/RC}\right]$$

Now the current can be expressed as follows,

$$V = V_R + V_C$$

$$V = i \cdot R + V_c$$

$$iR = V - V_c$$

$$iR = V - \left[V - Ve^{-t/RC}\right]$$

$$\therefore i = \frac{V}{R} e^{-t/RC}$$

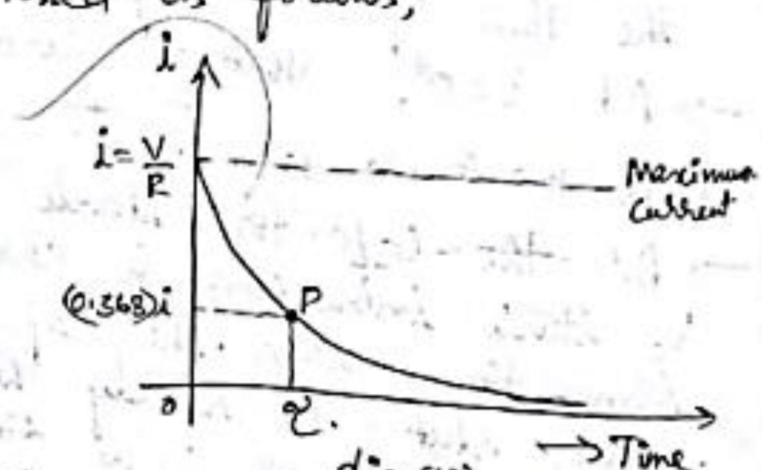


fig (10)

So at $t = 0$, $i = \frac{V}{R}$ is max current & in steady state it becomes zero.

DC Excitation to series R-L Circuit

i) D.C response of R-L series circuit

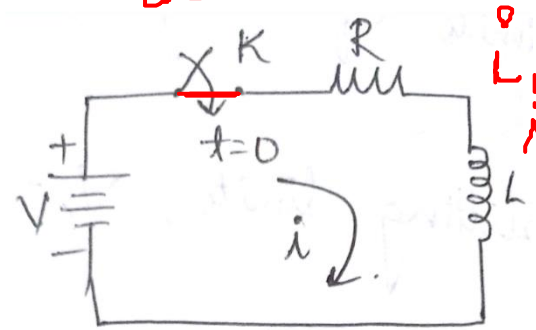
$i = (\quad)$

- At t=0-, $i_L(0^-) = 0$

Since the current through inductor cannot change instantaneously, $i_L(0^+) = 0$

- Let initial current be I_0. Here I_0 = 0.
- Assume switch K is closed at t=0
- After closing the switch, apply KVL

$V - Ri - L\frac{di}{dt} = 0$



$i_L(0^-) = 0$

$i_L(0^+) = 0$

$$\text{V} = \text{iR} + \text{L}\frac{di}{dt}$$

$\div$ by R

$$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}$$

$\frac{V}{R} - i = \frac{L}{R}\frac{di}{dt}$

$$\left(\frac{V}{R} - i\right) dt = \frac{L}{R}\, di$$

$\frac{R}{L} dt = \frac{di}{\left(\frac{V}{R} - i\right)}$

$$\left(\frac{R}{L}\right) dt = \frac{di}{\left(\frac{V}{R} - i\right)}$$

$i_L(t) = \frac{V}{R}$

Integrating both sides

$\int 1\, dx = x$

$\int \frac{1}{x} dx = \ln x$

$\frac{R}{L} t = -\ln\left(\frac{V}{R} - i\right) + K'$

$$\int \frac{R}{L}\, dt = \int \frac{di}{\left(\frac{V}{R} - i\right)}$$

$\frac{R}{L}\, t$ = -ln$\left[\frac{V}{R} - i\right] + K^{|}$ -------(1)

To find $K^{|}$, At t = 0, i = I_0 = 0.

0 = -ln $\left(\frac{V}{R}\right) + K^{|}$

$$\frac{R}{L} \cdot 0 = -\ln\left(\frac{V}{R} - 0\right) + K^{|}$$

Therefore, $K^{|}$ = ln $\left(\frac{V}{R}\right)$ --------(2)

Substitute equation (2) in (1), we get

$\frac{R}{L}$t = -ln $\left[\frac{V}{R}\text{-i}\right]$ + ln $\left[\frac{V}{R}\right]$

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

$$\frac{R}{L} t = \ln\left(\frac{\frac{V}{R}}{\frac{V}{R} - i}\right)$$

$$\frac{R}{L}\, t = \ln\left[\frac{\frac{V}{R}}{\left(\frac{V}{R} - i\right)}\right]$$

Taking antilog, $e^{\left(\frac{R}{L}\right)t} = \frac{V/R}{\left(\frac{V}{R} - i\right)}$

$\frac{V}{R} - i = \frac{V}{R} \, . \, e^{-\left(\frac{R}{L}\right)t}$

$i = \frac{V}{R} - \frac{V}{R} \, . \, e^{-\left(\frac{R}{L}\right)t}$

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$i = \frac{V}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$

$\frac{V}{R}$ is steady state part

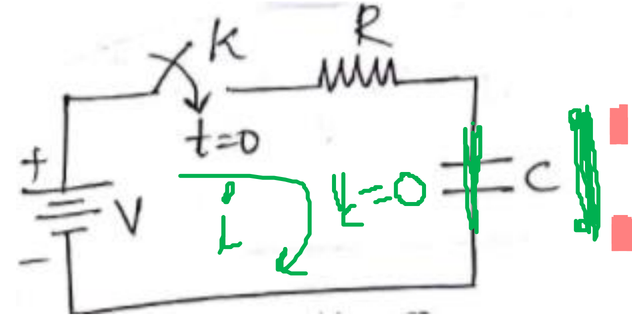
$\frac{V}{R} \, . \, e^{-\left(\frac{R}{L}\right)t}$ is transient part.

DC Excitation to series R-C Circuit

i) D.C response of R-C series circuit

→ Initially switch K is in open state so no charge on condenser.

→ At $t=0$, K is closed, capacitor acts as short circuit, so current at the time of switching is high. ∴ $V_c = 0$



$$i = \frac{V}{R}$$

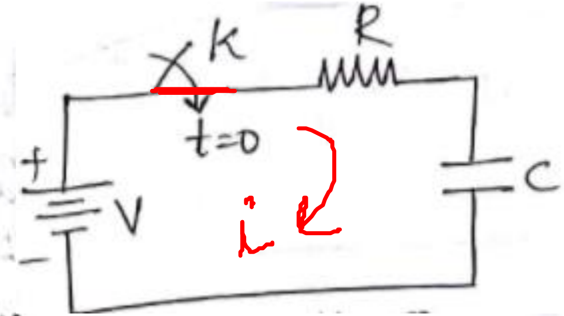
→ At $t=0^+$, voltage across capacitor $= 0$.
i is maximum at $t=0^+$.

→ As the capacitor starts charging, the voltage across capacitor V_c starts increasing & charging current starts decreasing. After some time, when the capacitor charges to V volts, it achieves steady state. In steady state, it acts as a open circuit so current will be zero finally.

→ After switching instant, applying KVL, $V = V_R + V_c$ $\quad V - V_R - V_c = 0$

$$V = i \cdot R + V_c$$

But current i can be written as $i = C\dfrac{dV_c}{dt}$



$$V = \left(C\frac{dV_c}{dt}\right)R + V_c$$

$V_c = (\quad)$

$$V - V_c = RC\frac{dV_c}{dt}$$

$$(V - V_c)\,dt = (RC)\,dV_c$$

$$\int \frac{1}{RC}\,dt = \int \frac{dV_c}{(V - V_c)}$$

$$\int \frac{dt}{RC} = \int \frac{dV_c}{(V - V_c)}$$

$$\frac{1}{RC}t = -\ln(V - V_c) + k' \longrightarrow (1)$$

$$i_c = C\frac{dV_c}{dt}$$

To find k'; At $t = 0$; $V_c = 0$.

$$0 = -\ln V + k'$$

$$\therefore k' = \ln[V]$$

$\log a - \log b = \log\left(\frac{a}{b}\right)$

Substitute k' in eqn (1).

$$\frac{1}{RC}t = -\ln(V - V_c) + \ln[V]$$

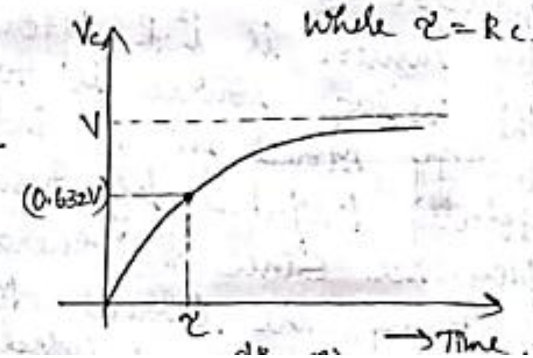
$$\frac{t}{RC} = \ln\left[\frac{V}{V - V_c}\right]$$

Taking antilog, $e^{(1/RC)t} = \dfrac{V}{V - V_c}$

$$V - V_c = V e^{-t/RC}$$

$$\therefore V_c = V(1 - e^{-t/RC})$$

where $\tau = RC$.



$$(V - V_c) = \frac{V}{e^{(1/RC)t}}$$

$$(V - V_c) = Ve^{-\left(\frac{1}{RC}\right)t}$$

$$V - Ve^{-t/RC} = V_c \quad ; \quad V_c = V\left(1 - e^{-t/RC}\right)$$

When the steady state is achieved, total charge on the capacitor is Q coulombs.

$\therefore V = \frac{Q}{C}$

III^{ly} at any instant, $V_c = \frac{q}{C}$, where q is instantaneous charge

$$\frac{q}{C} = \frac{Q}{C}\left[1 - e^{-t/RC}\right]$$

$$\therefore \underline{q = Q\left[1 - e^{-t/RC}\right]}$$

Now the current can be expressed as follows,

$$V = V_R + V_C$$

$$V = i \cdot R + V_C$$

$$iR = V - V_C$$

$$iR = V - \left[V - V e^{-t/RC}\right]$$

$$\therefore \underline{i = \frac{V}{R} e^{-t/RC}}$$

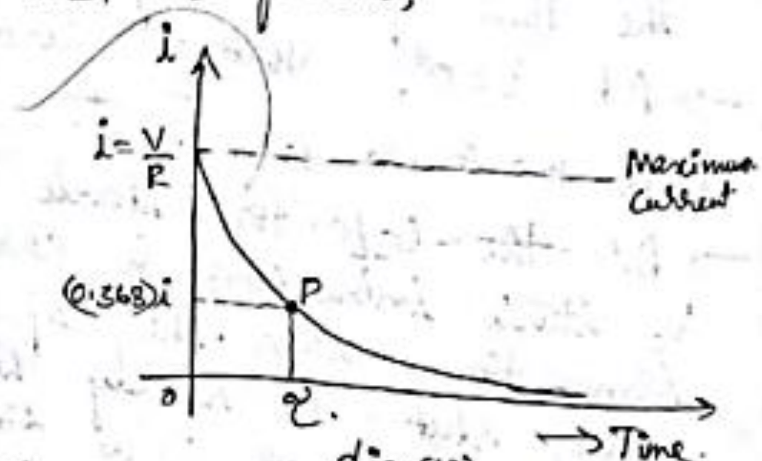
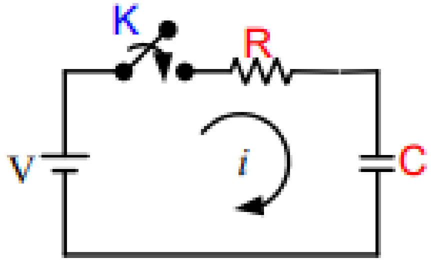


fig (10).

So at $t = 0$, $i = \frac{V}{R}$ is max current & in steady state it becomes zero.

Q 1) In the circuit shown in Figure 1.9 the switch K s closed at t=0, with capacitor uncharged. Find the values i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at t=0+, for element values as follows V=100 V $R = 1000\ \Omega$ and $C = 1\ \mu F$.

Solution:



KVL for the given circuit is

$$V = Ri + \frac{1}{C}\int idt \qquad (1.1)$$

At t=0+ the capacitor acts as short circuit which is as shown in Figure 1.10

Differentiating equation (1)

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

Substituting initial conditions

$$\frac{di}{dt}(0+) = -\frac{i(0+)}{RC}$$

$$0 = R\frac{di}{dt}(0+) + \frac{i(0+)}{C}$$

$$\frac{di}{dt}(0+) = -\frac{0.1}{1000 \times 1 \times 10^{-6}} - 100A/sec$$

$$\frac{di}{dt}(0+) = -\frac{i(0+)}{RC}$$

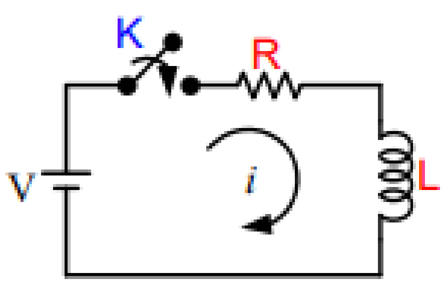
$$\frac{d^2i}{dt^2}(0+) = -\frac{1}{RC}\frac{di}{dt}(0+)$$

$$= -\frac{.1}{1000 \times 1 \times 10^{-6}}(-100)$$

$$= -\frac{.1}{1000 \times 1 \times 10^{-6}}(-100)$$

$$= 1 \times 10^5 A/sec^2$$

Q 2) In the circuit shown in Figure 1.11 the switch K s closed at t=0, with zero current in the conductor. Find the values i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at t=0+, for element values as follows V=100 V $R = 10\ \Omega$ and $L = 1\ H$.

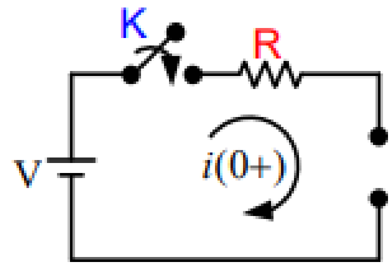


Solution:

$$V = Ri + L\frac{di}{dt}$$

at t=0+ the inductor acts as open circuit which is as shown in Figure 1.12

$$i(0+) = 0$$



$$L\frac{di}{dt} = V - Ri$$

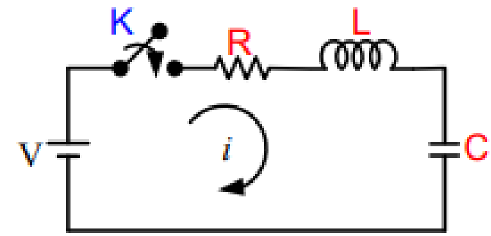
$$L\frac{di}{dt}(0+) = V - Ri(0+) = 100 - 0$$

$$\frac{di}{dt}(0+) = \frac{100}{1} = 100A/sec$$

$$\frac{d^2i}{dt^2}(0+) = -\frac{R}{L}\frac{di}{dt}(0+) = -\frac{10}{1} \times 100$$

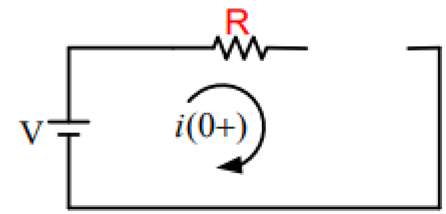
$$= -1000A/sec^2$$

Q 3) In the circuit shown in Figure 1.13 V=10 v $R = 10\ \Omega$ $L = 1\ H$ and $C = 10\ \mu F$ and $v_c(0) = 0$, find $i(0+) = 0$, $\frac{di}{dt}(0+)$ and $\frac{d^2i}{dt^2}(0+)$.



Solution:

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0$$



at t=0+ the inductor acts as open circuit and capacitor acts as short circuit which is as shown in Figure 1.31

$$i(0+) = 0$$

$$\begin{aligned} V &= Ri(0+) + L\frac{di}{dt}(0+) + \frac{1}{C}\int i(0+)dt \\ V &= R \times 0 + L\frac{di}{dt}(0+) + 0 \\ L\frac{di}{dt}(0+) &= V \\ \frac{di}{dt}(0+) &= \frac{V}{L} = \frac{10}{1} = 10A/sec \end{aligned}$$

Differentiating equation (1)

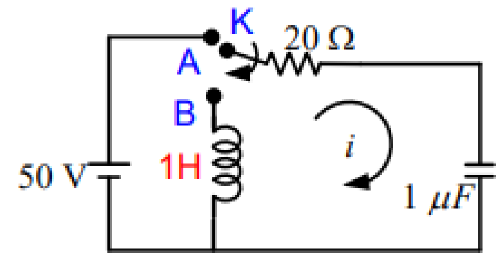
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$R\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} = 0$$

$$\begin{aligned} 10 \times 10 + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} &= 0 \\ \frac{d^2i}{dt^2}(0+) = \frac{-100}{L} = \frac{-100}{1} &= -100A/sec^2 \end{aligned}$$

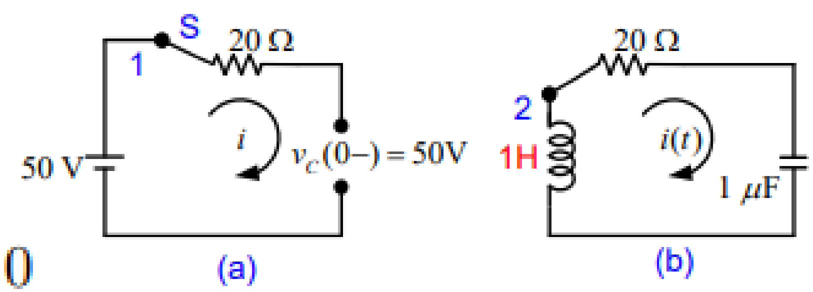
4. For the circuit shown in Figure 1.42 the switch K is changed from position A to B at t=0, the steady state having been reached before switching. Calculate i, $\frac{di}{dt}$, *and* $\frac{d^2i}{dt^2}$ at t=0+.



Before connecting to position 2 switch was at position 1 at t=0- under steady state condition capacitor charges with voltage of $v(0-) = 50 = v(0+)$ and after that it acts as an open circuit which is as shown in Figure 1.43 (a)

At t = 0- , inductor is in open circuit and capacitor is after fully charging it is also in open circuit state. That is

$$i(0-) = 0 \;\; and\; also \;\; i(0+) = 0$$



When switch is at position 2, and at t=0+ the circuit diagram is as shown in Figure 1.43 (b)

Applying KVL for the circuit we have

$$\begin{aligned} Ri + L\frac{di}{dt} + \frac{1}{C}\int idt &= 0 \\ Ri + L\frac{di}{dt} + v_c(t)dt &= 0 \end{aligned}$$

$$Ri + L\frac{di}{dt} + v_c(t)dt \quad = \quad 0$$

At t $= 0+$ and $v_c(0+) = 50$

$$\begin{aligned}
Ri(0+) + L\frac{di}{dt}(0+) + v_c(0+) &= 0 \\
20 \times 0 + 1\frac{di}{dt}(0+) + 50 &= 0 \\
\frac{di}{dt}(0+) &= \frac{-50}{1} = -50 \ A/sec
\end{aligned}$$

Differentiating equation (1)

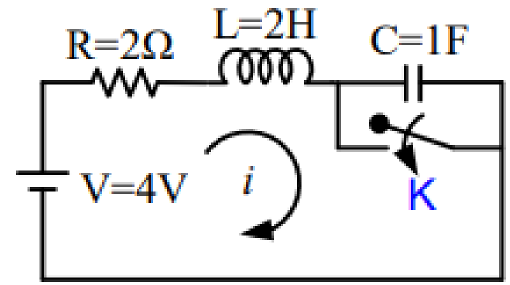
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned}
R\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} &= 0 \\
20 \times (-50) + 1\frac{d^2i}{dt^2}(0+) + \frac{0}{C} &= 0 \\
\frac{d^2i}{dt^2}(0+) &= \frac{1000}{1} = 1000A/sec^2
\end{aligned}$$

5. For the circuit shown in Figure 1.38 the switch K is opened at t=0. Find i $\frac{di}{dt}, v_c, \frac{dv_c}{dt}$ at t=0+.

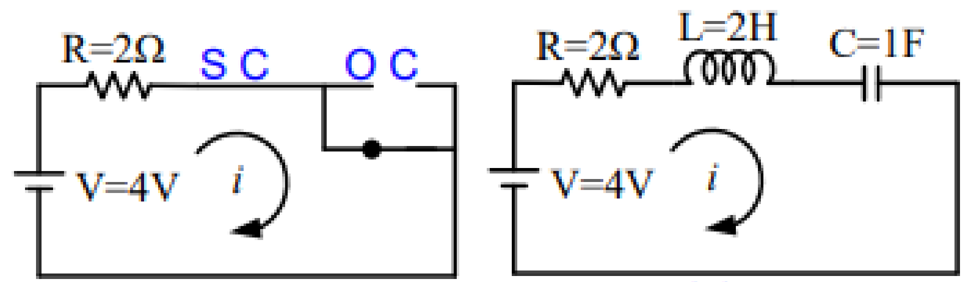
Solution: When switch is at position a at t=0- circuit which is as shown in Figure 1.39



When switch is at opened, at t=0+ circuit which is as shown in Figure 1.39 (a)

$$V = Ri(0-)$$

$$i(0-) = \frac{V}{R} = \frac{4}{2} = 2\ A$$



(a) (b)

When switch is at opened, at t=0+ circuit which is as shown in Figure 1.39 (b)
Applying KVL

$$V = \frac{1}{C}\int i dt + L\frac{di}{dt} + Ri$$

At t=0+

$$2 = 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+)$$

$$\begin{aligned} 2 &= 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+) \\ \frac{di}{dt}(0+) &= 2 - 1 \times 2 = 0 \end{aligned}$$

The voltage across capacitor is

$$v_c(t) = \frac{1}{C}\int i$$

$$\frac{dv_c(t)}{dt} = \frac{1}{C}i$$

$$\frac{dv_c(t)}{dt}(0+) = \frac{1}{C}i(0+)$$

$$\frac{dv_c(t)}{dt}(0+) = \frac{1}{1}2 = 2V$$

When switch is opened at t=0+ circuit diagram is as shown in Figure 1.41 (b)

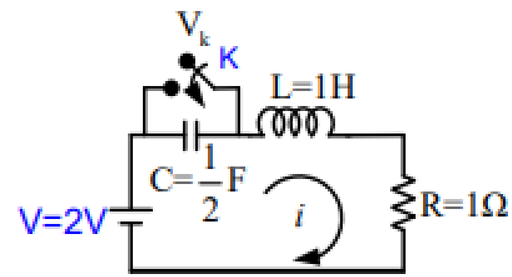
Applying KVL

$$V = \frac{1}{C}\int idt + L\frac{di}{dt} + Ri$$

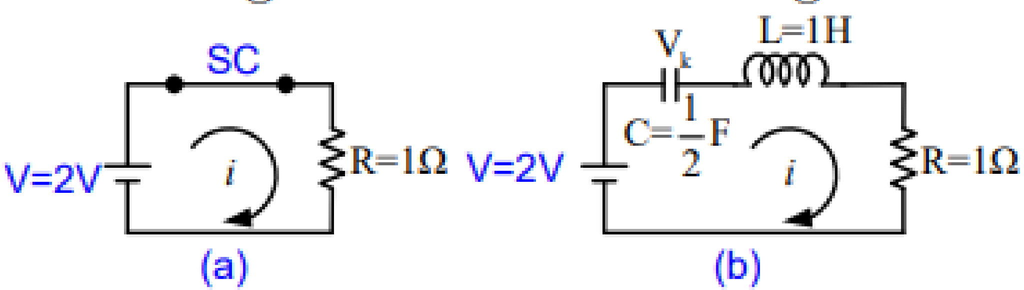
At t=0+

$$2 = 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+)$$

6. For the circuit shown in Figure 1.40 the switch K is opened at t=0 after reaching the steady state condition. Determine voltage across switch and its first and second derivatives at t=0+.



Solution: Before opening switch and at t=0- circuit diagram is as shown in Figure 1.41 (a)



$$2 = 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+)$$

$$\frac{di}{dt}(0+) = 2 - 1 \times 2 = 0$$

$$V = Ri(0-)$$

$$i(0-) = \frac{V}{R} = \frac{2}{1} = 2\ A = i(0+)$$

$$V = \frac{1}{C}\int idt + L\frac{di}{dt} + Ri$$

Differentiating above Equation

$$\begin{aligned} 0 &= \frac{i}{C} + L\frac{d^2i}{dt^2} + R\frac{di}{dt} \\ 1 \times \frac{d^2i_1}{dt^2}(0+) &= -R\frac{di}{dt}(0+) - \frac{i(0+)}{C} \\ \frac{d^2i(0+)}{dt^2} &= -1 \times 0 - \frac{2}{1/2}(0+) = -4\ A/sec^2 \end{aligned}$$

$$V = \frac{1}{C}\int idt + L\frac{di}{dt} + Ri$$

Differentiating above Equation

$$\begin{aligned} 0 &= \frac{1}{C}\frac{di}{dt} + L\frac{d^3i}{dt^3} + R\frac{d^2i}{dt^2} \\ 1 \times \frac{d^3i_1}{dt^3}(0+) &= -R\frac{d^2i(0+)}{dt^2} - \frac{1}{C}\frac{d(0+)i}{dt} \\ \frac{d^3i(0+)}{d3^2} &= -1 \times (-4) - 0 = 4\ A/sec^2 \end{aligned}$$

The voltage across capacitor is

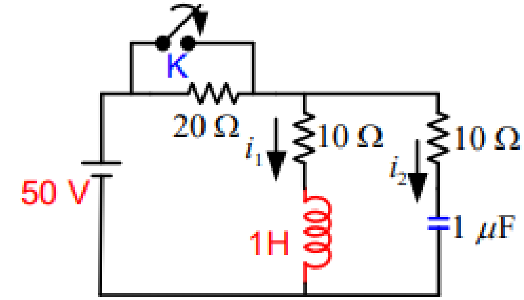
$$V_k + L\frac{di}{dt} + R \times i = 2$$

$$\begin{aligned} \frac{dV_k}{dt} + L\frac{d^2i}{dt^2} + R \times \frac{di}{dt} &= 0 \\ \frac{dv_c(t)}{dt}(0+) &= \frac{1}{C}(5-5) = 0 \end{aligned}$$

$$\frac{d^2V_k}{dt^2} + L\frac{d^3i}{dt^3} + R \times \frac{d^2i}{dt^2} = 0$$

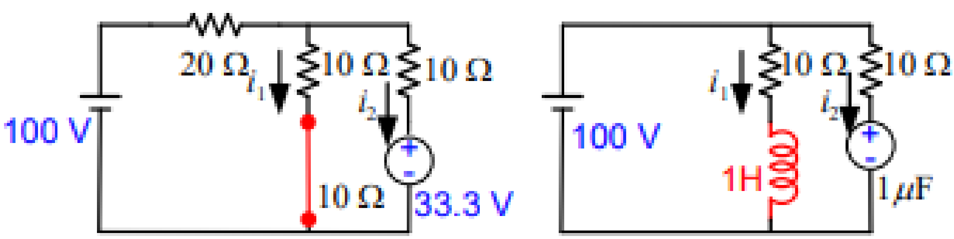
$$\begin{aligned} \frac{d^2V_k(0+)}{dt^2} &= -L\frac{d^3i(0+)}{dt^3} + R \times \frac{d^2i(0+)}{dt^2} \\ \frac{d^2V_k(0+)}{dt^2} &= -1 \times 4 - 1 \times (-4) = 0\ V/sec^2 \end{aligned}$$

7. In the circuit shown in Figure the steady state is reached with switch K is open. The switch K is closed at t=0. Solve for $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}$ at t=0+.



Solution:

When switch is opened and when steady state is reached capacitor acts as open circuit and inductor acts as short circuit which is as shown in Figure 1.45(a).



(a) (b)

$$i_1(0-) = \frac{V}{20+10} = \frac{100}{30} = 3.33A$$

$$i_2(0-) = 0A$$

Voltage across capacitor is voltage across R_2

$$v_c(0-) = i_1(0-) \times 10 = 3.33 \times 10 = 33.33V$$

When switch is closed at t=0 20Ω is short circuited. Inductor acts as current source with a value of 3.33 A

and capacitor acts as voltage source with a value of 33.33 V which is as shown in Figure 1.45(b).

$$i_2(0+) = \frac{100 - 33.33}{10} = 6.667$$

$$V = 10i_1 + L\frac{di_1}{dt}$$

$$V = 10i_1(0+) + L\frac{di_1(0+)}{dt}$$

$$
\begin{aligned}
1\frac{di_1(0+)}{dt} &= 100 - 10 \times 3.33 \\
\frac{di_1(0+)}{dt} &= 66.7A/sec
\end{aligned}
$$

For the capacitor branch

$$V = 10i_2 + \frac{1}{C}\int i_2 dt$$

$$V = 10i_2(0+) + \frac{1}{C}\int i_2(0+)dt$$

Differentiating we get

$$
\begin{aligned}
&= 10\frac{di_2(0+)}{dt} + \frac{1}{C}i_2(0+) \\
\frac{di_2(0+)}{dt} &= -\frac{1}{10 \times C}i_2(0+) \\
&= -\frac{1}{10 \times 1 \times 10^{-6}} \times 6.667 = 0.667 \times 10^6 \ A/sec
\end{aligned}
$$