A T M E College of Engineering

Department of EEE
Emitting Elite Energy

Department of Electrical & Electronics Engineering

BEE302 – ELECTRIC CIRCUIT ANALYSIS

Outline:

Module 1 – Basic Concepts:

- *Introduction.*
- *Active and passive elements.*
- *Concept of ideal and practical sources.*
- *Concept of independent and dependent sources.*

Introduction

Circuit

Interconnection of 2 or more simple circuit elements forms an electrical circuit.

Analysis

Process:

- ➢ Determine the scope of the problem.
- ➢ Obtain the information required to understand it.
- ➢ Compute the parameters of the interest.

Basic Definitions:

1. ***Network***: The interconnection of two or more simple circuit elements forms an electrical network.
2. ***Circuit:*** The network containing atleast one closed path is an electric circuit.
3. ***Network Element:*** Any individual circuit element with two terminals which can be connected other circuit element is called a network element.

Network / Circuit elements can be either active elements or passive elements.

Active elements are the elements which supply power or energy to the network.

Eg:- Voltage source, Current Source.

Passive elements are the elements which either store energy or dissipate energy in the form of heat.

Eg: Resistor, inductor & Capacitor

4. Linear Network: A Circuit or network whose parameters i.e., elements like resistances, inductances & capacitances are always constant irrespective of the change in time, voltage, temperature etc is known as linear network.

5. Non – linear Network: - A circuit whose parameters change their values with change in time, temperature, voltage etc is known as non linear network.

A network consisting of an element like diode whose characteristics depends on temperature is an example of non linear network.

6. Bilateral Network: A circuit whose characteristics behavior is same irrespective of the direction of current through various elements of it, is called bilateral network.

Eg:- Network consisting only **resistances** is a bilateral network.

7. Unilateral Network: A circuit whose operation, behavior is dependent on the direction of the current through various elements is called unilateral network.

Eg: - Circuit consisting **diodes**, which allows flow of current only in one direction is an unilateral network.

8. Active Network: A circuit which contains atleast one source of energy is called active network.

Voltage source, current source, transistors, op-amps, batteries, signal generators are the examples of active elements.

9. Passive Network: A circuit which contains no energy source is called passive circuit. These networks consist of passive elements only.

Eg:- **Resistors,inductors,capacitors,thermistors etc**

10. DC Network:- A network consisting of dc sources which are fixed polarity sources, not varying with time is called a DC network.

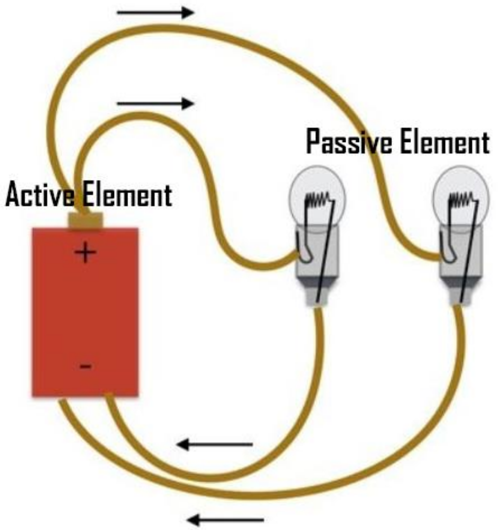
11. AC Network :- A network consisting of ac sources which are alternating sources periodically varying with time, is called an AC network.

12. Lumped Network :- A network in which all the network elements are physically separable is known as lumped network.

13. Distributed Network:- A network in which the circuit elements like resistance, inductance etc cannot be physically separable for analysis purposes is called distributed network.

Eg:- **Transmission Line.**

Active & Passive Components



The Two Types of Electronic Devices

Electronic elements that make up a circuit are connected together by conductors to form a complete circuit.

Active components

Passive components

Active Components

An **active component** is an electronic component which supplies energy to a circuit.

Common examples of active components include:

Voltage sources

Current sources

Generators (such as alternators and DC generators)

All different types of ***transistors*** (such as bipolar junction transistors, MOSFETS, FETs, and JFET)

Diodes (such as Zener diodes, photodiodes, Schottky diodes, and LEDs)

Voltage Sources

- A voltage source is an example of an active component in a circuit.
- When current leaves from the positive terminal of the voltage source, energy is being supplied to the circuit.
- A battery can also be considered as an active element, as it continuously delivers energy to the circuit during discharging.

Current Sources

- A current source is also considered an active component. As it is controlling the flow of charge in a circuit
- The current supplied to the circuit by an ideal current source is independent of circuit voltage.

Transistors

Transistors are also an active circuit component, because transistors are able to amplify the power of a signal.

Passive Components

- A **passive component** is an electronic component which can only receive energy, which it can either dissipate, absorb or store it in an electric field or a magnetic field.
- Passive elements do not need any form of electrical power to operate.

As the name ‘passive’ suggests – passive devices do not provide gain or amplification.

Passive components cannot amplify, oscillate, or generate an electrical signal.

Common examples of passive components include:

Resistors

Inductors

Capacitors

Transformers

A resistor is taken as a passive element since it can not deliver any energy to a circuit. Instead resistors can only receive energy which they can dissipate as heat as long as current flows through it.

An inductor is also considered as passive element of circuit, because it can store energy in it as a magnetic field, and can deliver that energy to the circuit, but not in continuous basis. The energy absorbing and delivering capacity of an inductor is limited and transient in nature.

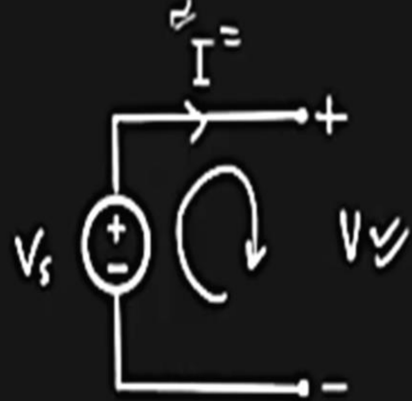
A capacitor is considered as a passive element because it can store energy in it as electric field. The energy dealing capacity of a capacitor is limited and transient – it is not actually supplying energy, it is storing it for later use.

A transformer is also a passive electronic component. Although this can seem surprising since transformers are used to raise voltage levels – remember that power is kept constant.

When transformers step up (or step down) voltage, power and energy remain the same on the primary and secondary side.

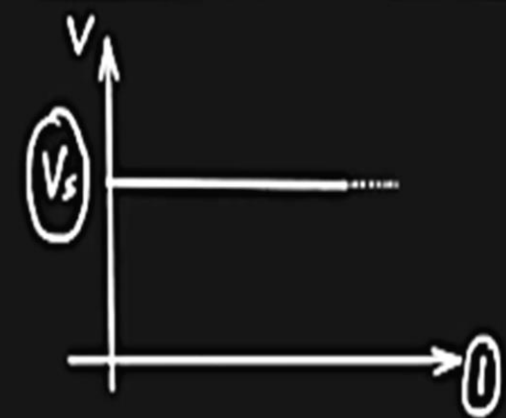
Ideal and Practical Voltage Sources

Ideal Voltage Source: has zero internal resistance and it delivers the energy at a specified voltage, which doesn't depend on the current delivered by the source.



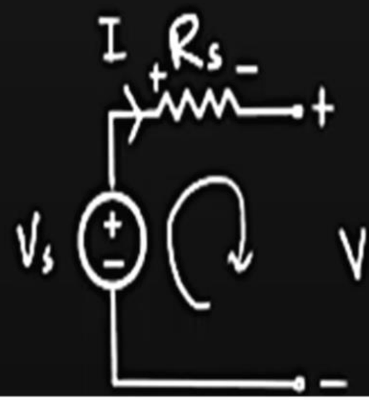
$$+V_s - V = 0$$

$$V = V_s$$



$R_s \neq 0$

Practical Voltage Source: has a finite internal resistance and it delivers the energy at a specified voltage, which depends on the current delivered by the source.



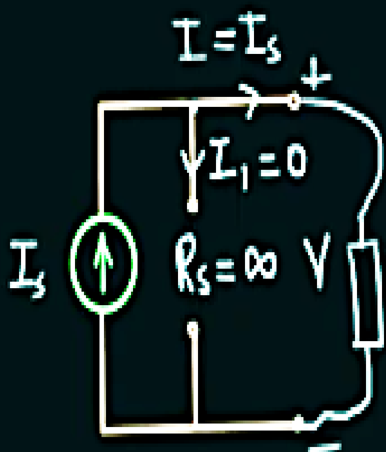
$$+V_s - IR_s - V = 0$$

$$\downarrow V = V_s - \uparrow \underbrace{IR_s}_{\approx 0}$$

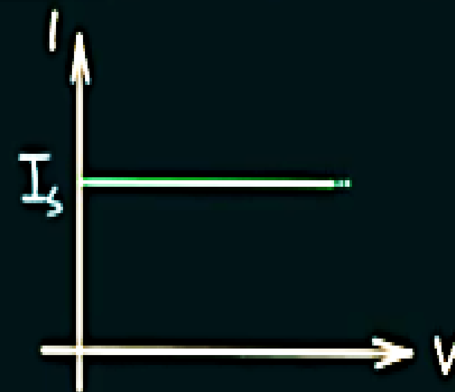
Ideal and Practical Current Sources

$R_s = \infty\ \Omega$

Ideal Current Source: has infinite internal resistance and it delivers the energy at a specified current which is independent of the voltage across the source.

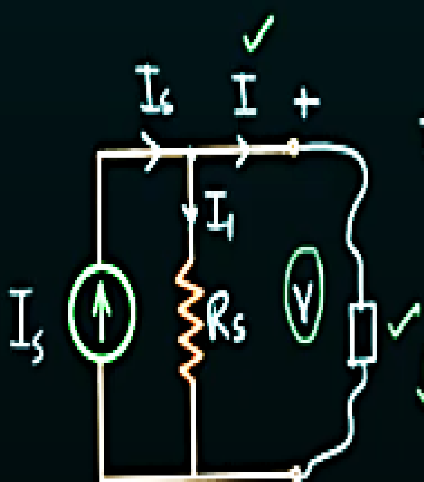


$I = I_s$



$R_s \neq \infty$

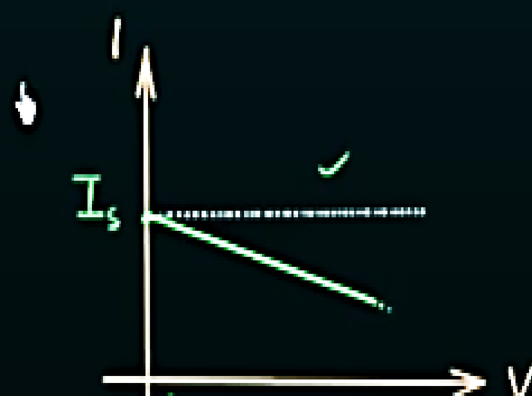
Practical Current Source: has finite internal resistance and it delivers the energy at a specified current which is dependent on the voltage across the source.



$I_s = I + I_1$

$I = I_s - I_1$

$I = I_s - \frac{V}{R_s}$

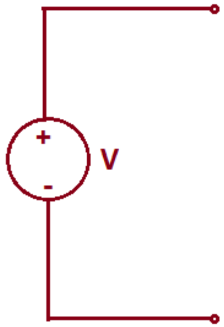


Concept of ideal and practical sources.

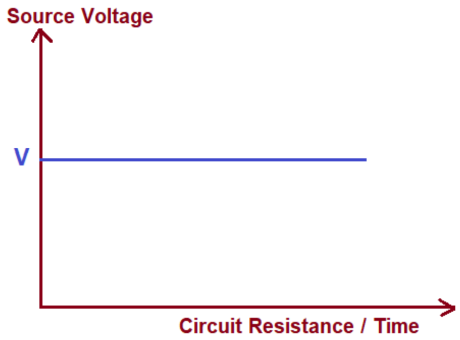
Ideal Voltage Source

An ideal voltage source is defined as the two terminal device capable of providing a constant voltage across its terminals. The voltage across the terminals of an ideal voltage source remains constant and is independent of load current.

Symbol:



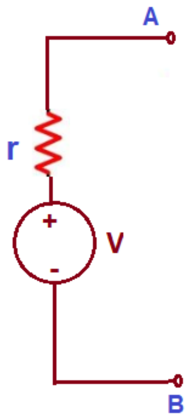
Characteristics:



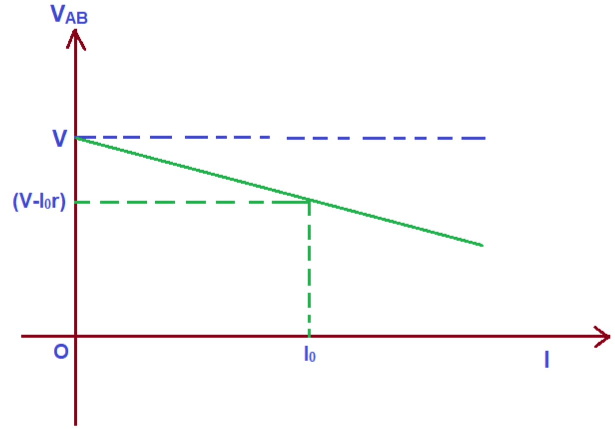
Practical Voltage Source:

- A practical voltage source is one which we find around us.
- The terminal voltage across its terminals are not constant rather it varies with output current.
- Internal resistance of practical voltage source has some finite value.
- Examples of voltage sources are Batteries, Generators etc.

Symbol:



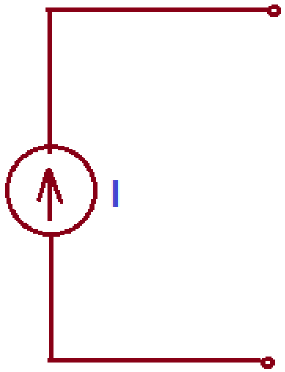
Characteristics:



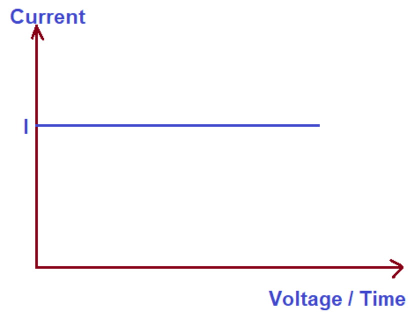
Ideal Current Source

- An ideal current source is a two terminal device which supplies constant current irrespective of load resistance.
- The value of current will be constant with respect to time and load resistance.
- This means that the power delivering capability is infinite for this source.
- An ideal current source has infinite parallel resistance connected to it.
- Therefore, the output current is independent of voltage of the source terminals.
- No such current source exists in the world, this is just a concept.
- However, every current source is designed to approach closer to the ideal one.

Symbol:



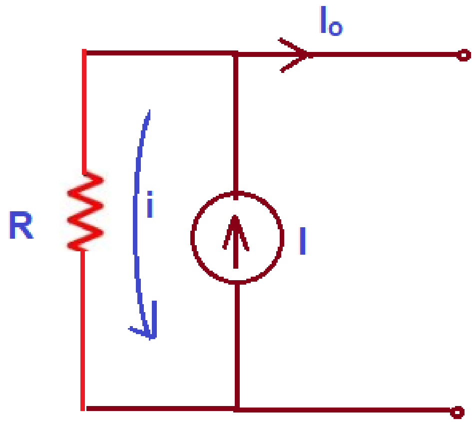
Characteristics:

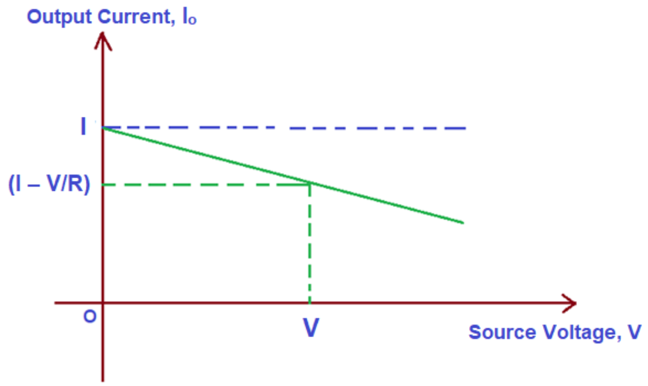


Practical Current Source:

- A practical current source is a two terminal device having some resistance connected across its terminals.
- Unlike ideal current source, the output current of practical source depends on the voltage of the source.
- The more this voltage, the lesser will be the current.

Symbol: **Characteristics:**



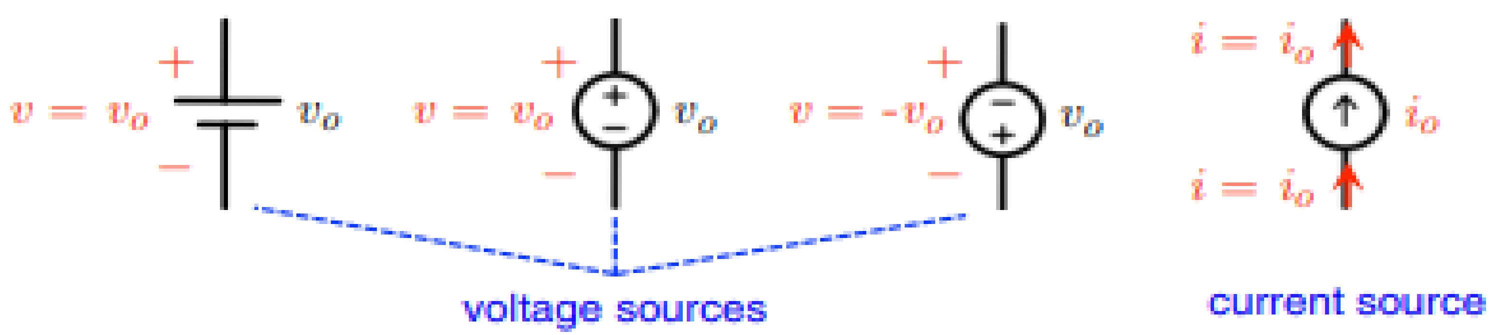


Independent Sources

- All circuits are excited with AC and DC sources.
- Among these, independent sources are defined as energy-delivering devices whose voltage or current values are fixed at a given value, independent of the rest of the circuit.
- Two types of independent sources are used in circuit analysis: *voltage and current sources*.
- An ideal voltage source is defined as

$$v(t) = v_0(t)$$

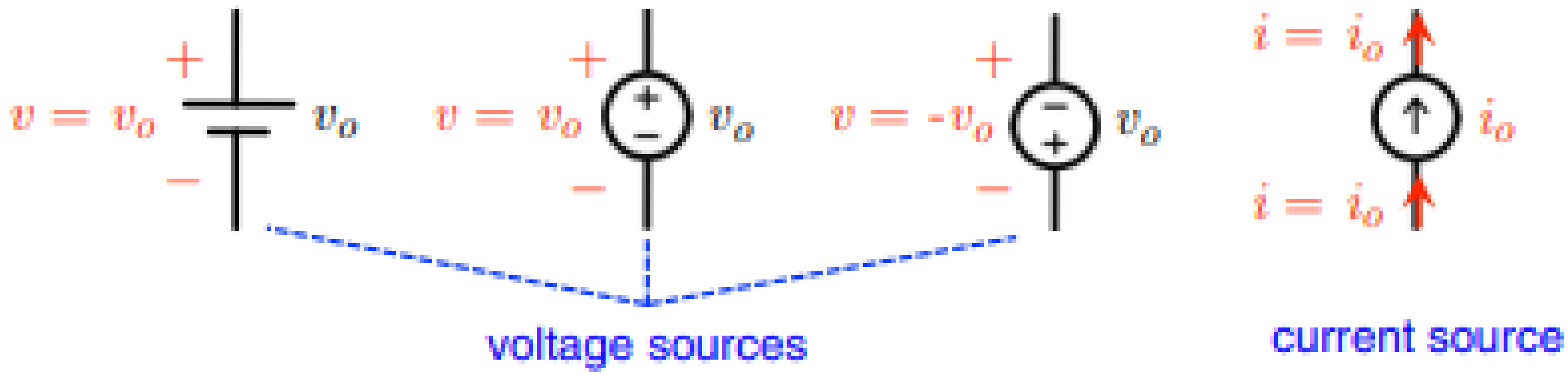
where $v_0(t)$ is given and independent of other parts of the circuit.



- If the voltage source is DC, we further have **$v(t) = v_0(t)$** as a constant.
- the current through a voltage source, i(t), can be anything (not necessarily zero).
- If a voltage source is delivering energy, i(t) must be nonzero.
- An ideal current source is defined as **$i(t) = i_0(t)$**

where $i_0(t)$ is given and independent of other parts of the circuit.

- If the current source is DC, we further have **$i(t) = i_0(t)$** as a constant.
- The voltage across a current source, $v(t)$, can be anything (not necessarily zero).

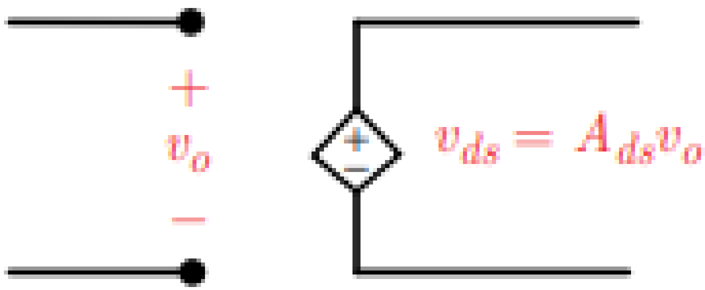


Dependent Sources

Dependent sources are also energy-delivering devices, where, unlike independent sources, the voltage or current provided depends on another voltage or current in the circuit.

Voltage-controlled voltage source (VCVS):

A voltage source whose voltage depends on another voltage in the circuit, i.e., $\boldsymbol{v_{ds} = A_{ds}\, v_o}$, where A_{ds} is a unit less quantity.



VCVS

Voltage-controlled current source (VCCS):

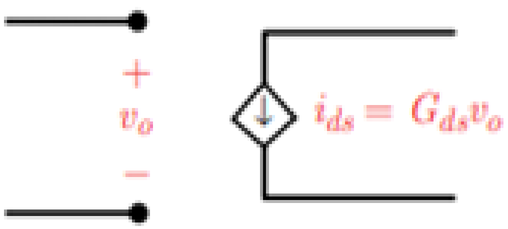
A current source whose current depends on a voltage in the circuit, i.e., $i_{ds} = G_{ds}v_0$, where G_{ds} is measured in siemens.

Current-controlled voltage source (CCVS):

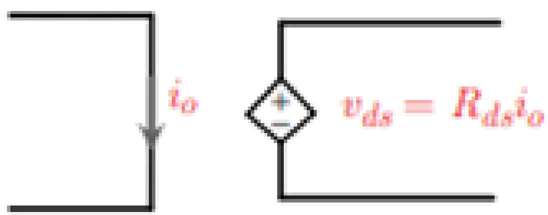
A voltage source whose voltage depends on a current in the circuit, i.e., $v_{ds}= R_{ds}i_0$, where R_{ds} is measured in ohms.

Current-controlled current source (CCCS):

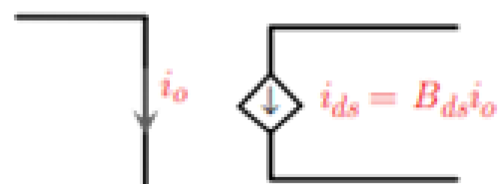
A current source whose current depends on another current in the circuit, i.e., $i_{ds} = B_{ds}\ i_0$, where B_{ds} is unit less.



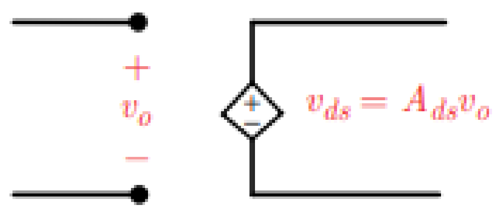
VCCS



CCVS



CCCS



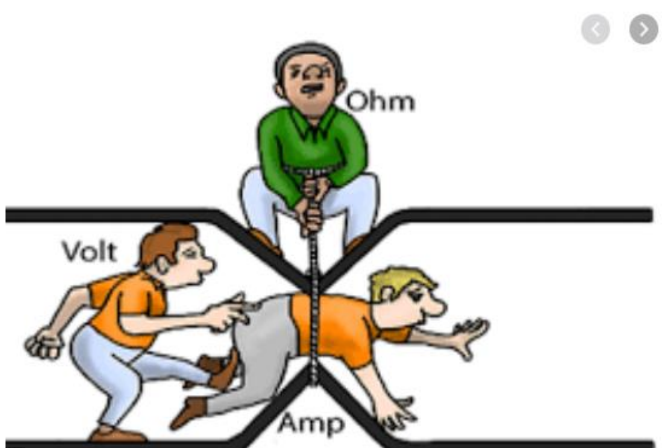
VCVS

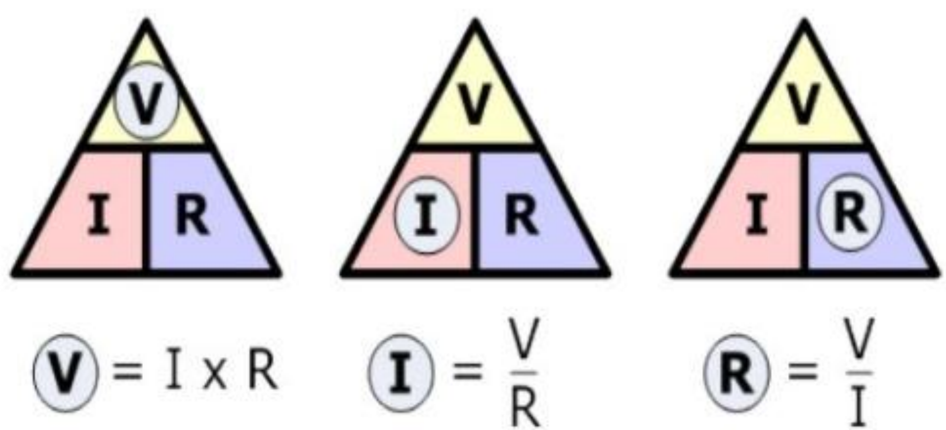
Ohm's Law

Statement:- " The Potential difference between the 2 ends of a conductor is directly proportional to the current flowing through it, provided its temperature & other physical parameters remain unchanged."

$$V \alpha I \qquad \text{or } V = IR$$

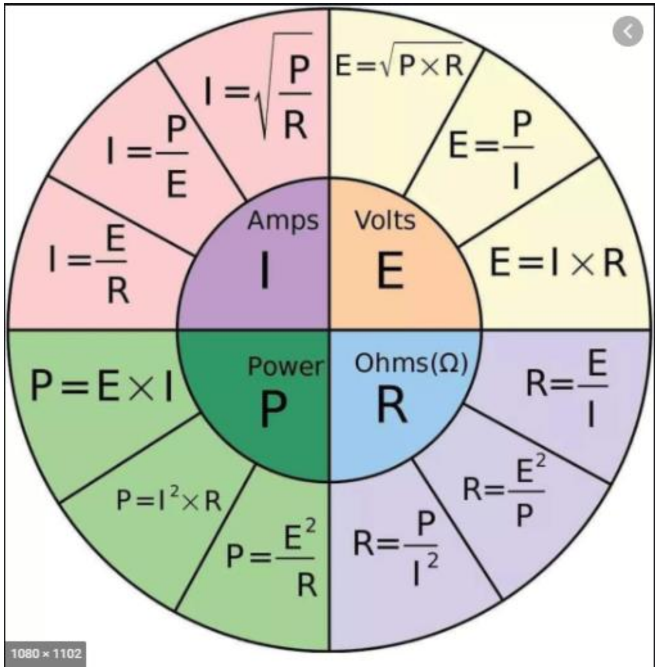
Where R is constant of proportionality called resistance.





Limitations

- It does not hold good for non-linear devices such as semiconductor & Zener diodes.
- Cannot be applied to arc-lamps
- It does not holds good where the temperature rise is rapid in some metals
- Not applicable to non-metallic conductors like silicon carbide.

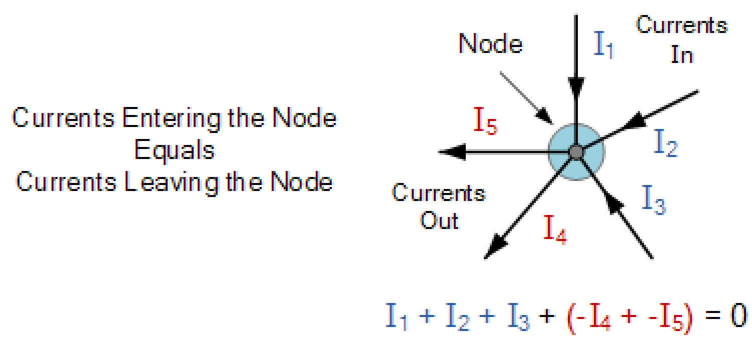


Kirchhoff's Laws

Kirchhoffs First Law – The Current Law, (KCL)

Kirchhoffs Current Law or KCL, states that the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

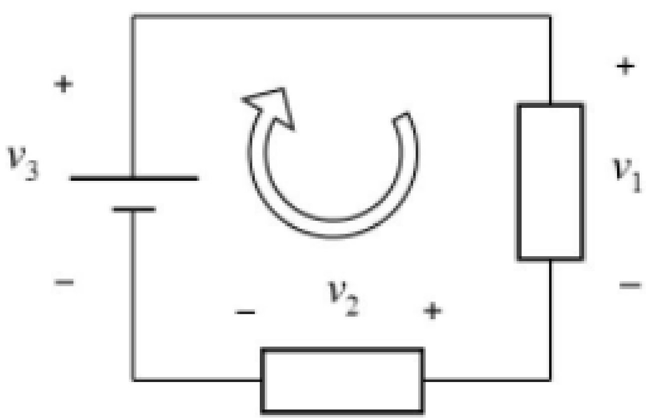
$$I_{(leaving)} + I_{(entering)} = 0$$



Kirchhoffs Second Law – The Voltage Law, (KVL)

Statement:- "In any closed circuit or mesh, the algebraic sum of products of currents & resistances (voltage drops) plus the algebraic sum of all the emf's in that circuit is zero,"

i.e, Algebraic sum of emf's + Algebraic sum of voltage drops = 0



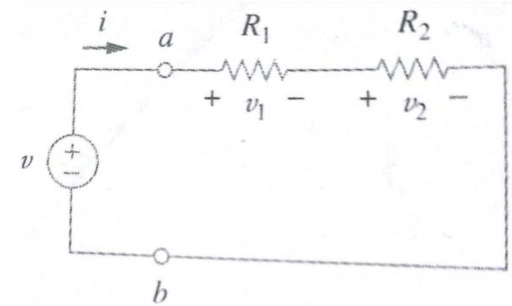
$$+v_1 + v_2 - v_3 = 0$$

$$-v_1 - v_2 + v_3 = 0$$

$$v_1 + v_2 = v_3$$

Series Circuit & Voltage Division

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.



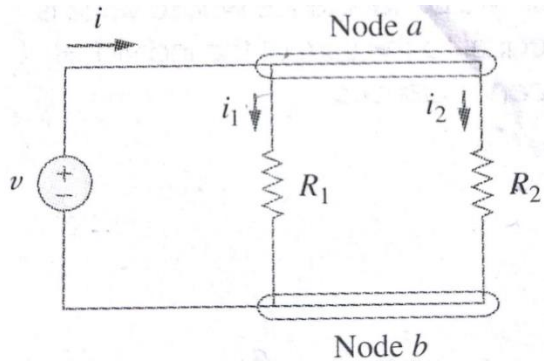
$$R_{eq} = R_1 + R_2$$

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

Parallel Circuit & Current Division

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

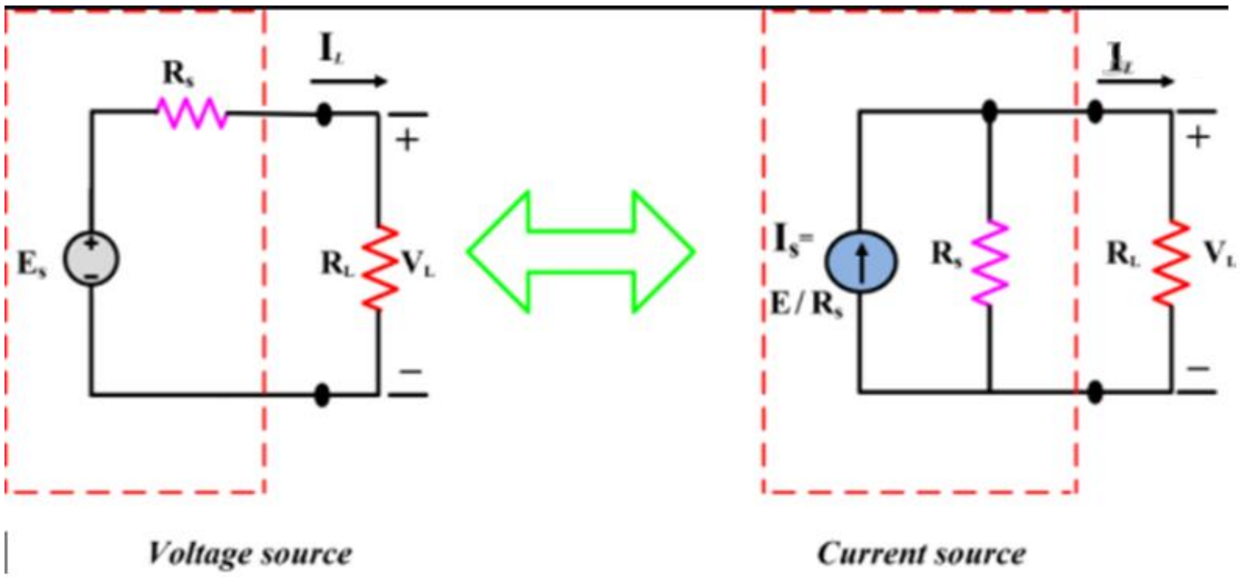
$$i_1 = \frac{R_2\, i}{R_1 + R_2}$$

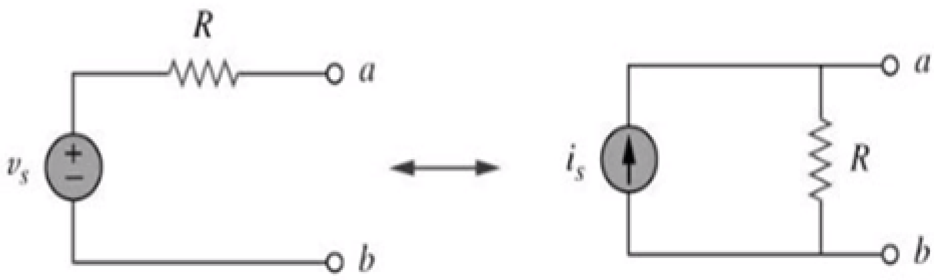
$$i_2 = \frac{R_1\, i}{R_1 + R_2}$$

Concept of Source Transformation

A Source transformation is the process of replacing *a voltage source V_S in series with a resistor R* by a *current source i_S in parallel with a resistor R*, or vice versa.

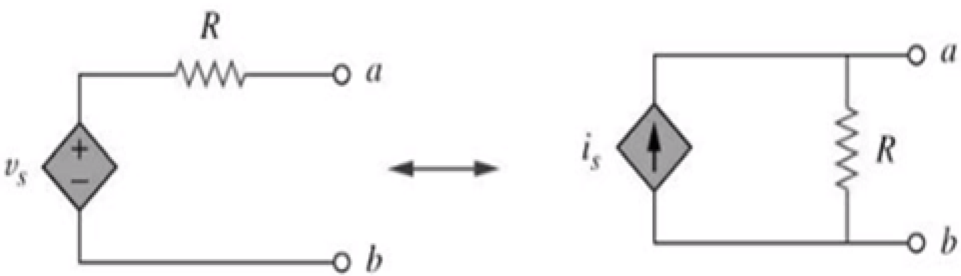
An equivalent circuit is one whose V-I characteristics are identical with the original circuit.





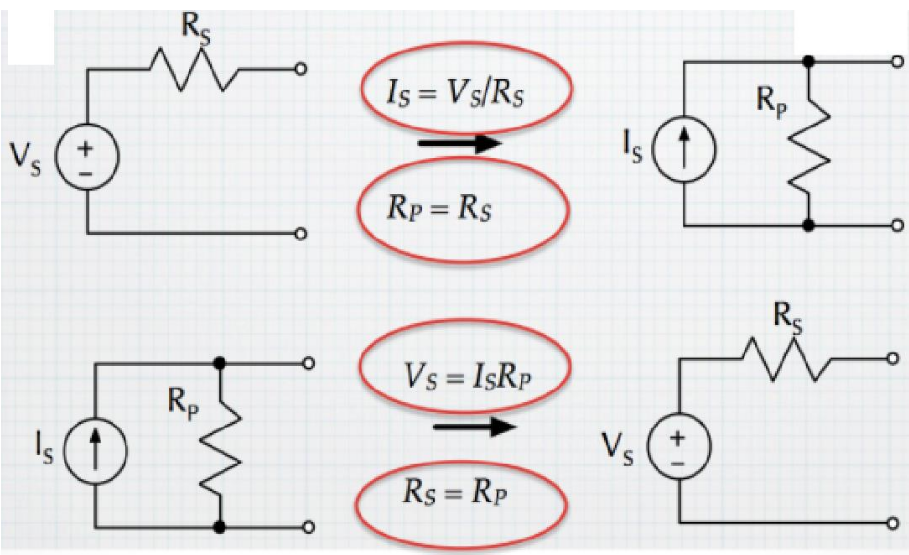
(a) Independent source transform

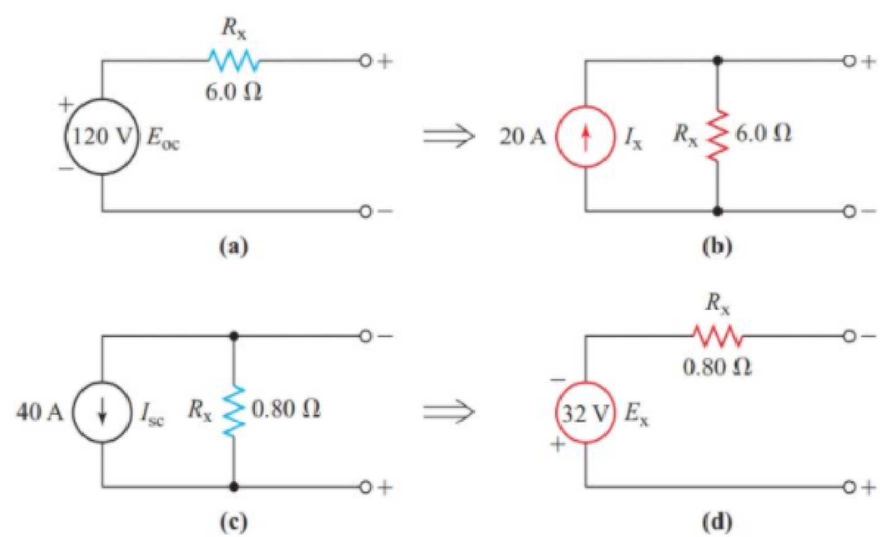
The arrow of the current source is directed toward the positive terminal of the voltage source.



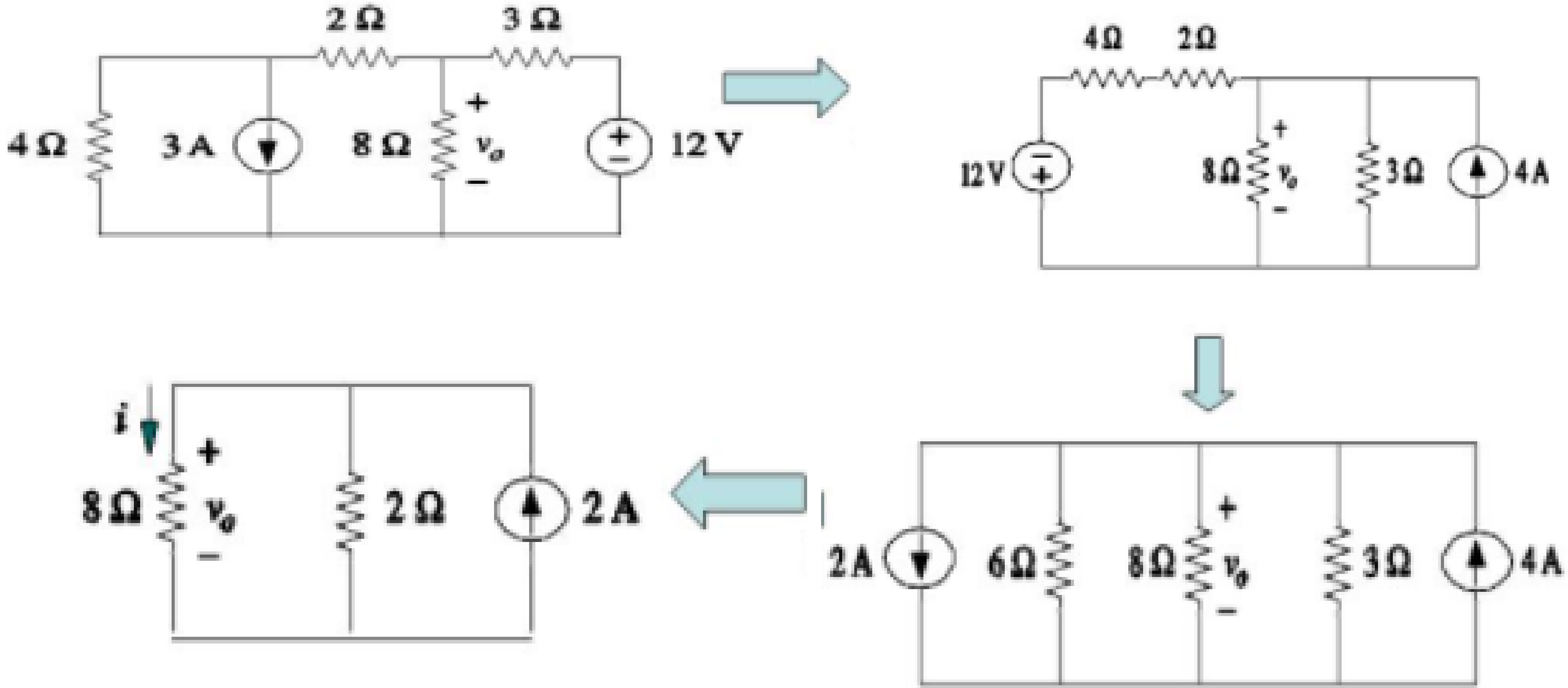
(b) Dependent source transform

The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source





1. Use source transformation to find v_0 in the circuit.

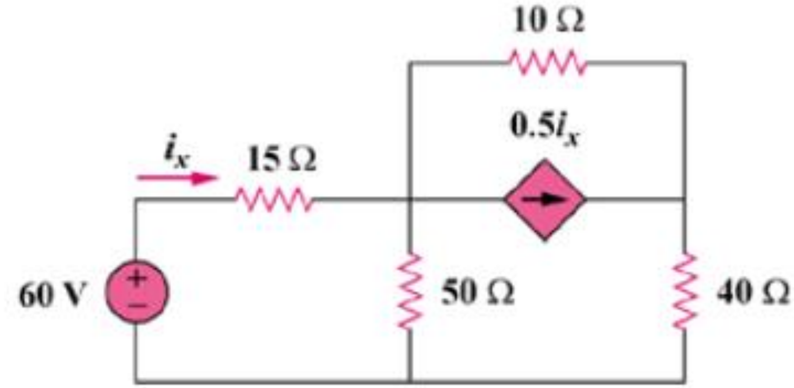


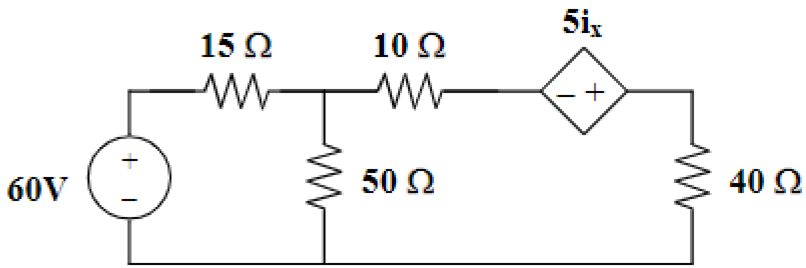
$$i = \frac{2}{8+2} * 2 = 0.4A$$

$$v_0 = 0.8 * 4 = 3.2\ V$$

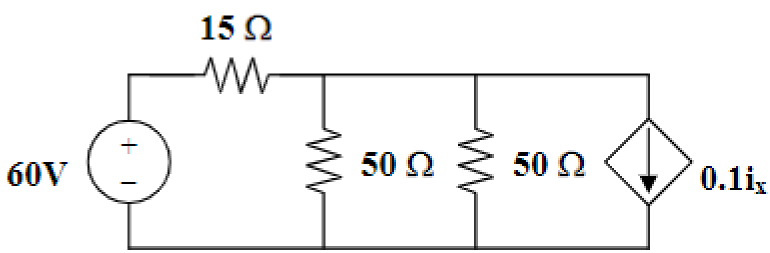
2. Use source transformation to find i(x) in the circuit of Fig.

Solution : As shown in Fig. (a), we transform the dependent current source to a voltage source,

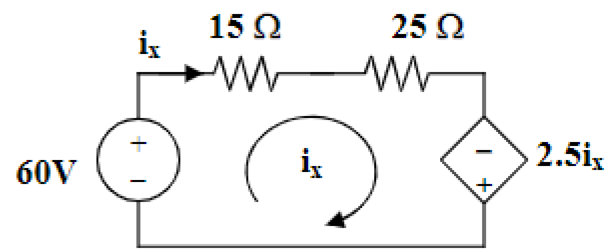




(a)



(b)



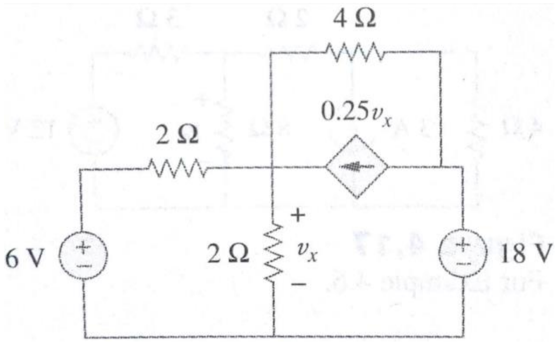
(c)

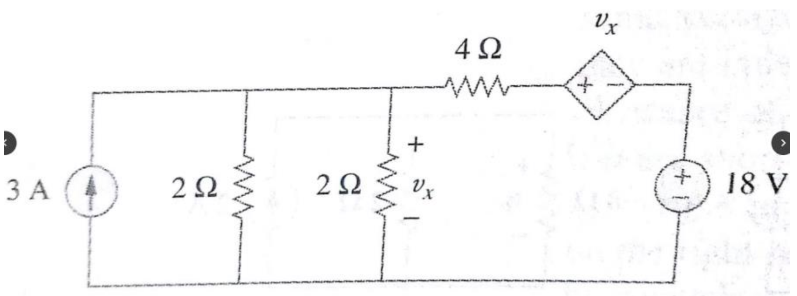
In Fig. (b), 50||50 = 25 ohms. Applying KVL in Fig. (c),

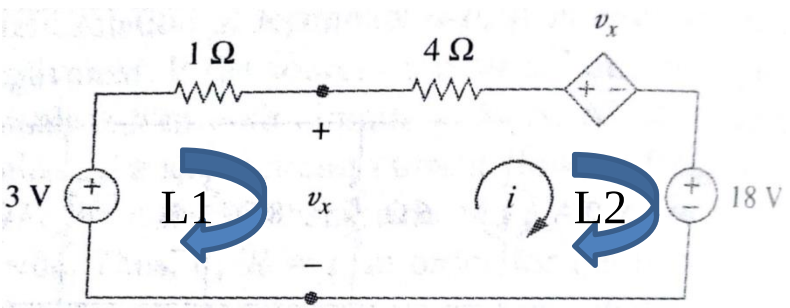
$$60 - 15\, i_x - 25\, i_x + 2.5 i_x = 0$$

i_x = **1.6 A**

3. Find v_x using source transformation.







Apply KVL to L1

$+3-1i- v_x = 0$

$v_x = 3 – i$

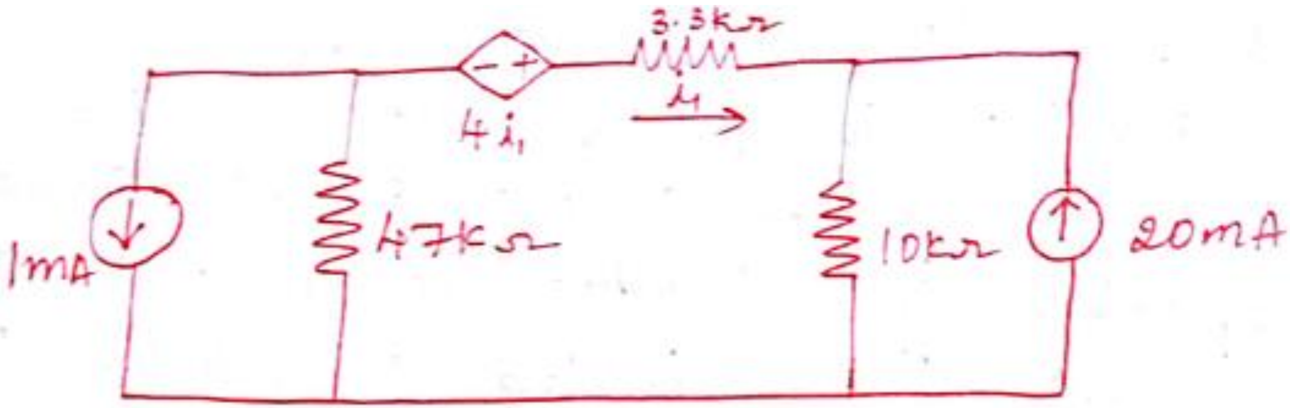
Apply KVL to L2

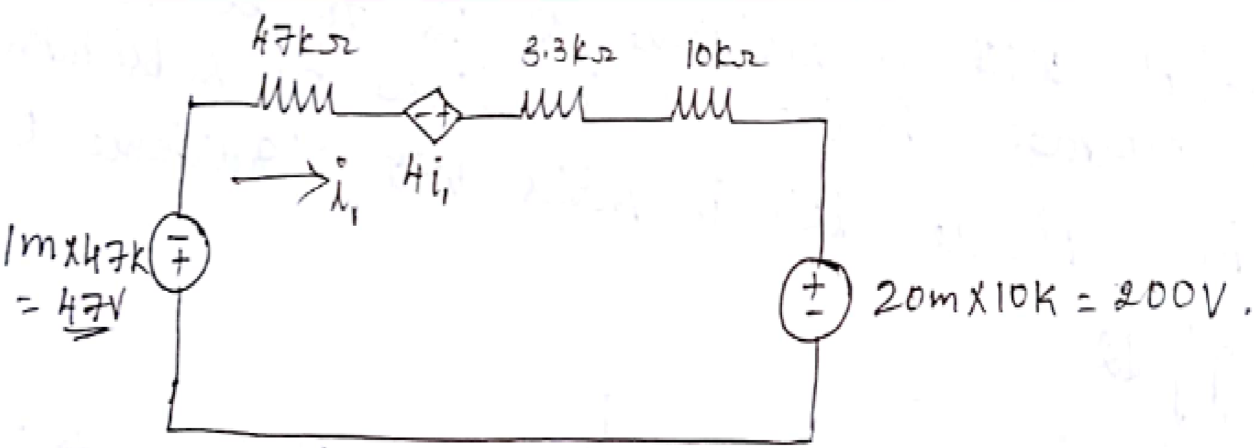
$-4i - v_x - 18 + v_x = 0$

$i = 18/(-4) = - 4.5A$

$v_x = 3 – (- 4.5) = 7.5$ V.

4. Find the current i_1 using source transformation for the circuit shown in the fig below





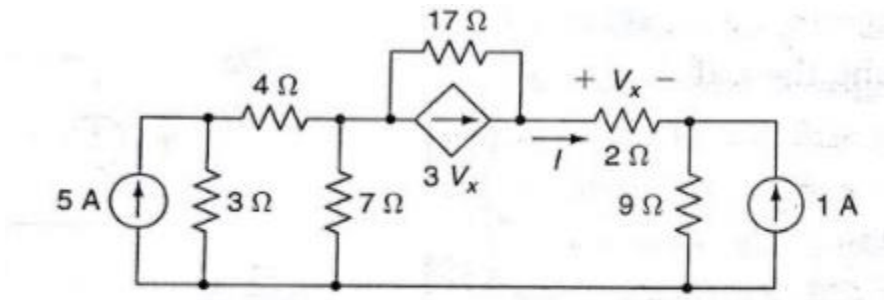
Applying KVL to above ckt.

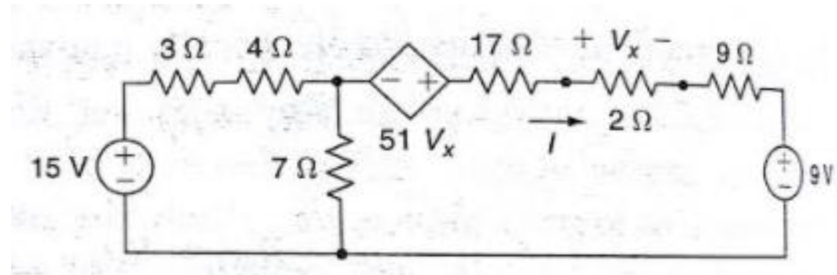
$$-47 - 47k\, i_1 + 4 i_1 - 13.3k\, i_1 - 200 = 0.$$

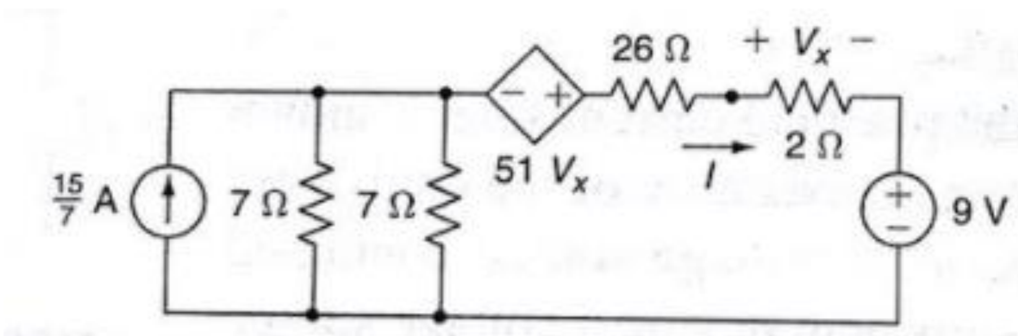
$$-247 - 60296\, i_1 = 0$$

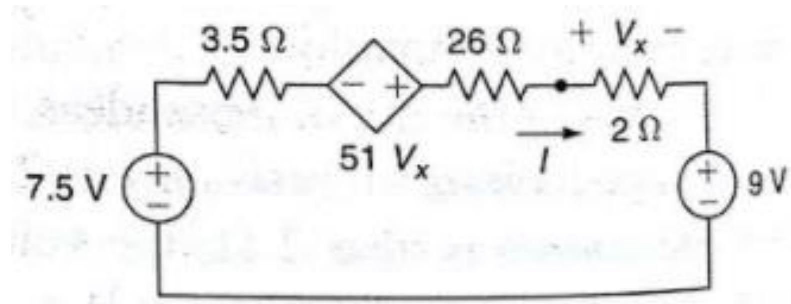
$$i_1 = 4.096 \text{ mA}$$

5. Calculate the current through the 2Ω resistor in fig below by making use of Source transformations to first simplify the circuit.









$7\cdot5-3\cdot5I+51V_x-26I-2I-9=$

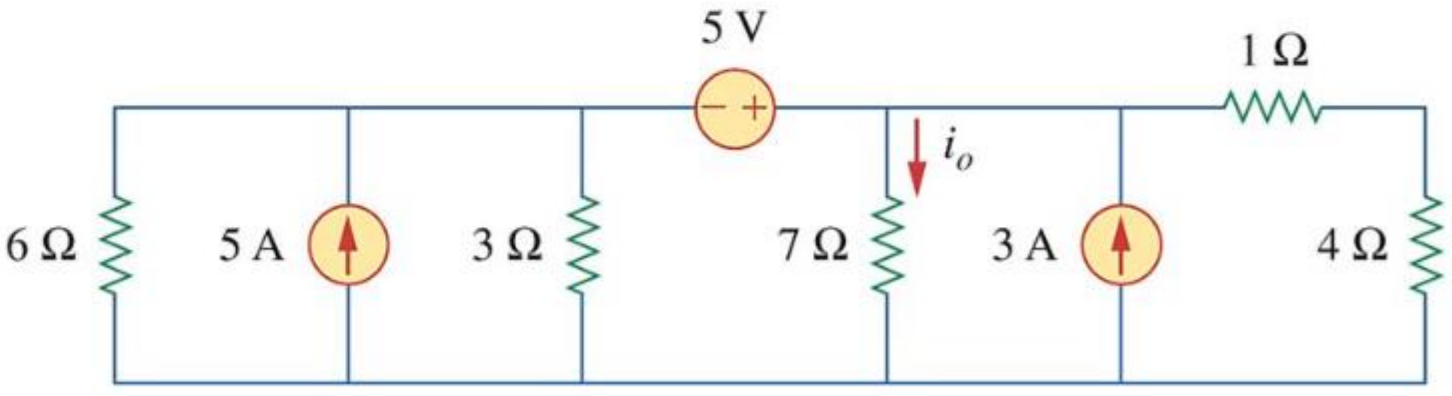
$$7.5 - 3.5I + 51V_x - 26I - 2I - 9 = 0$$

$$V_x = 2I$$

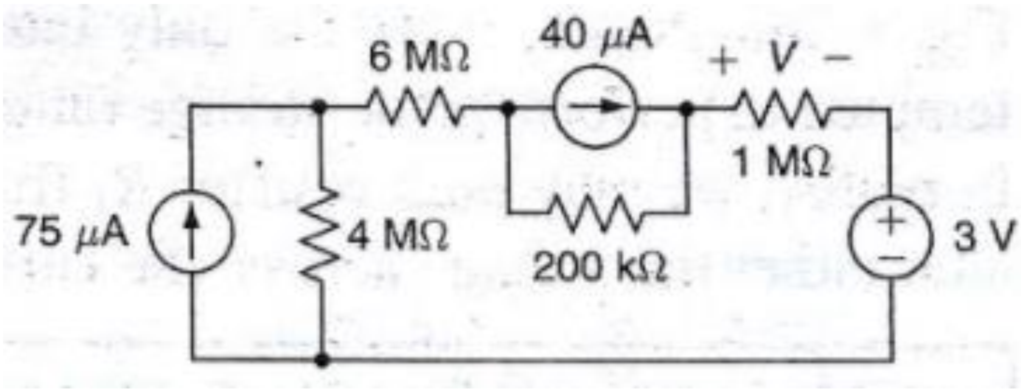
I = 21.28 mA

Practice Problems

1. Use source transformation to find i_0 in the circuit.



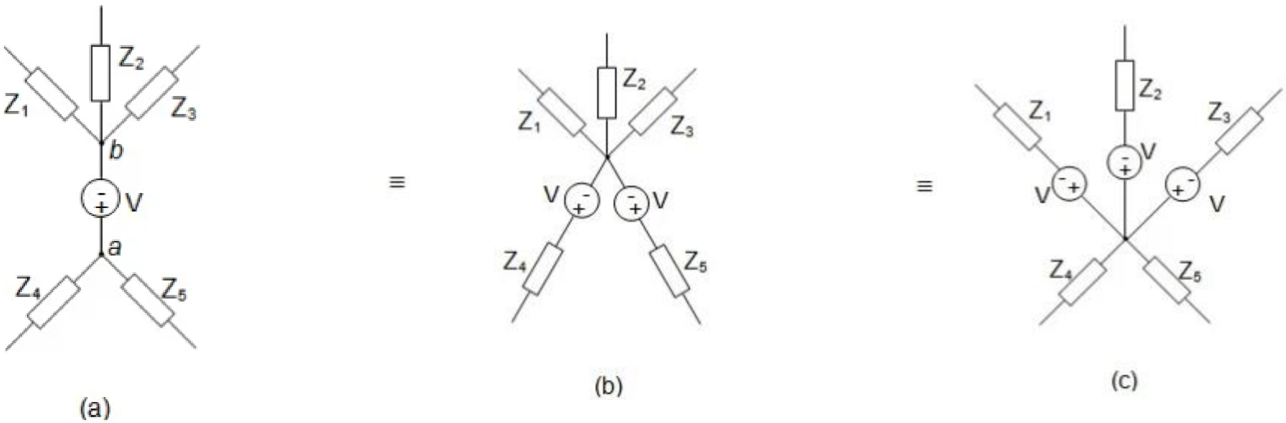
2. For the circuit of fig shown below, compute the voltage across the 1MΩ resistor using repeated source transformations.

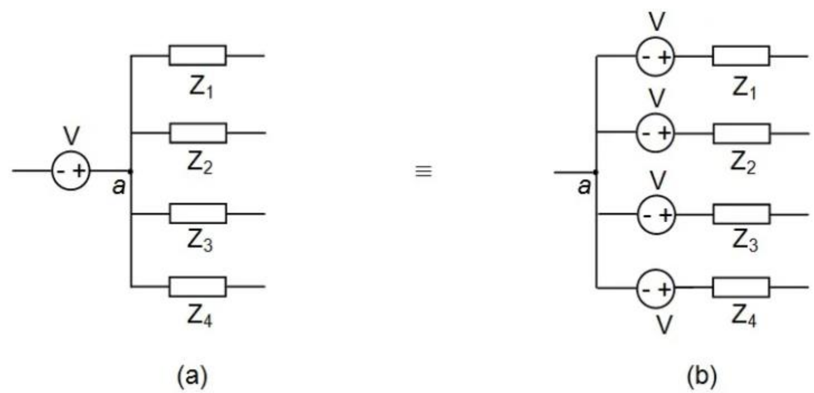


Concept of Source Shifting

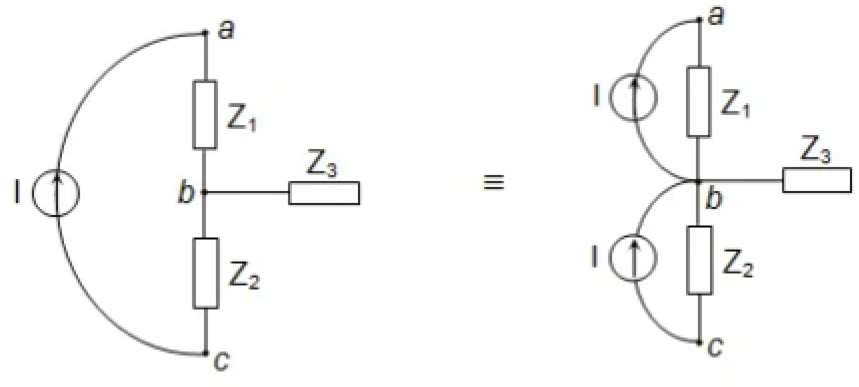
By source shifting, we may mean either the shifting of current source or shifting of the voltage source—very similar to that in the case of source transformation.

Shifting of Voltage Source (V-Shift)

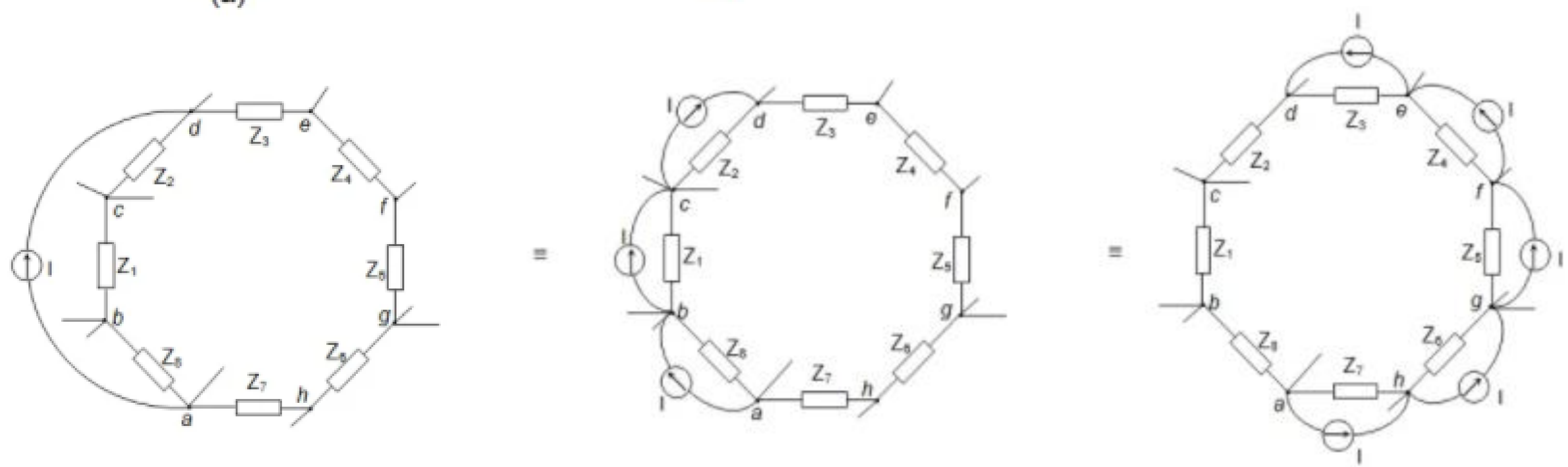




Shifting of Current Source (I-Shift)

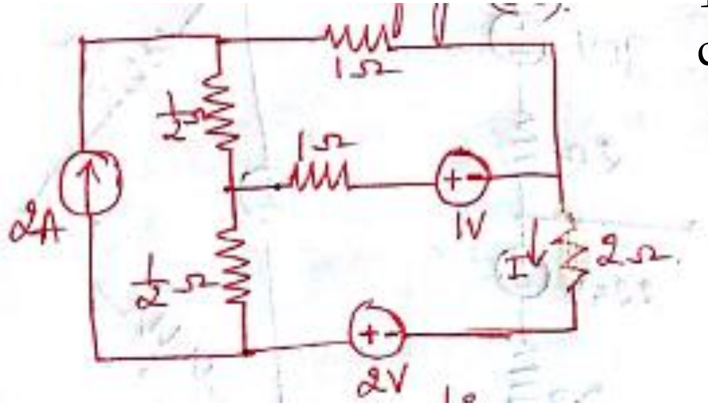


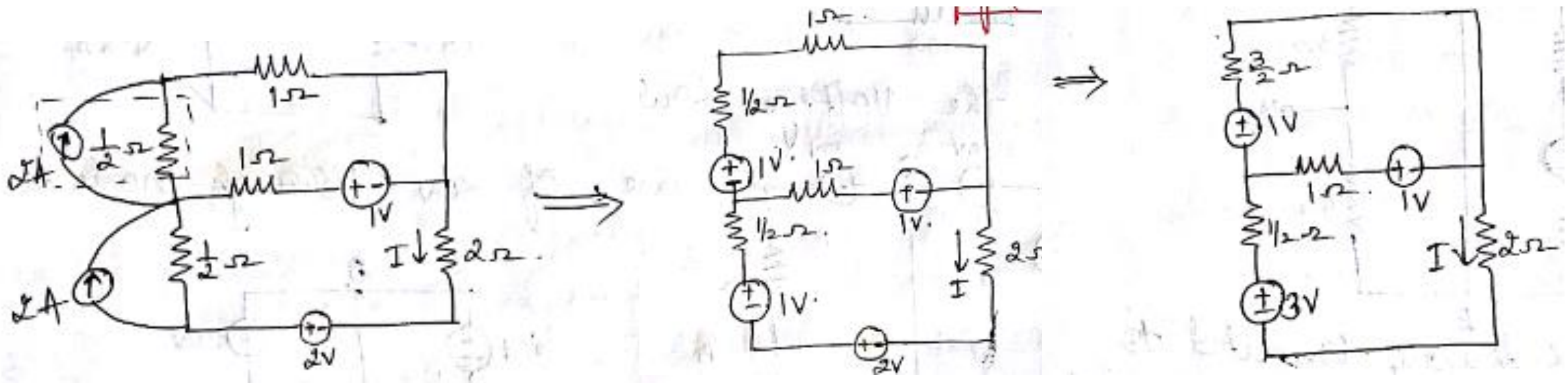
(a) (b)

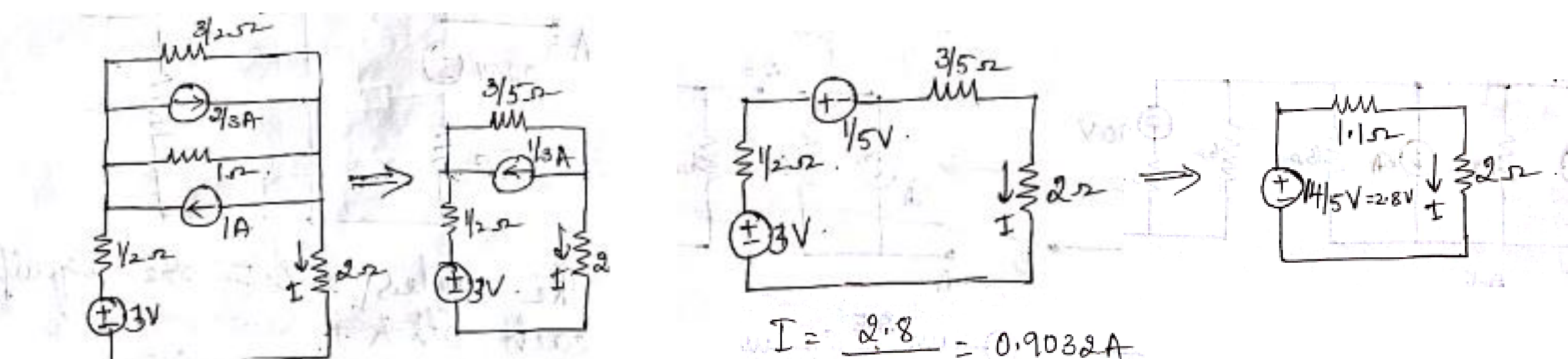


(a) (b) (c)

1. Using source shifting & source transformation, find current ‘I’ in the circuit shown in fig

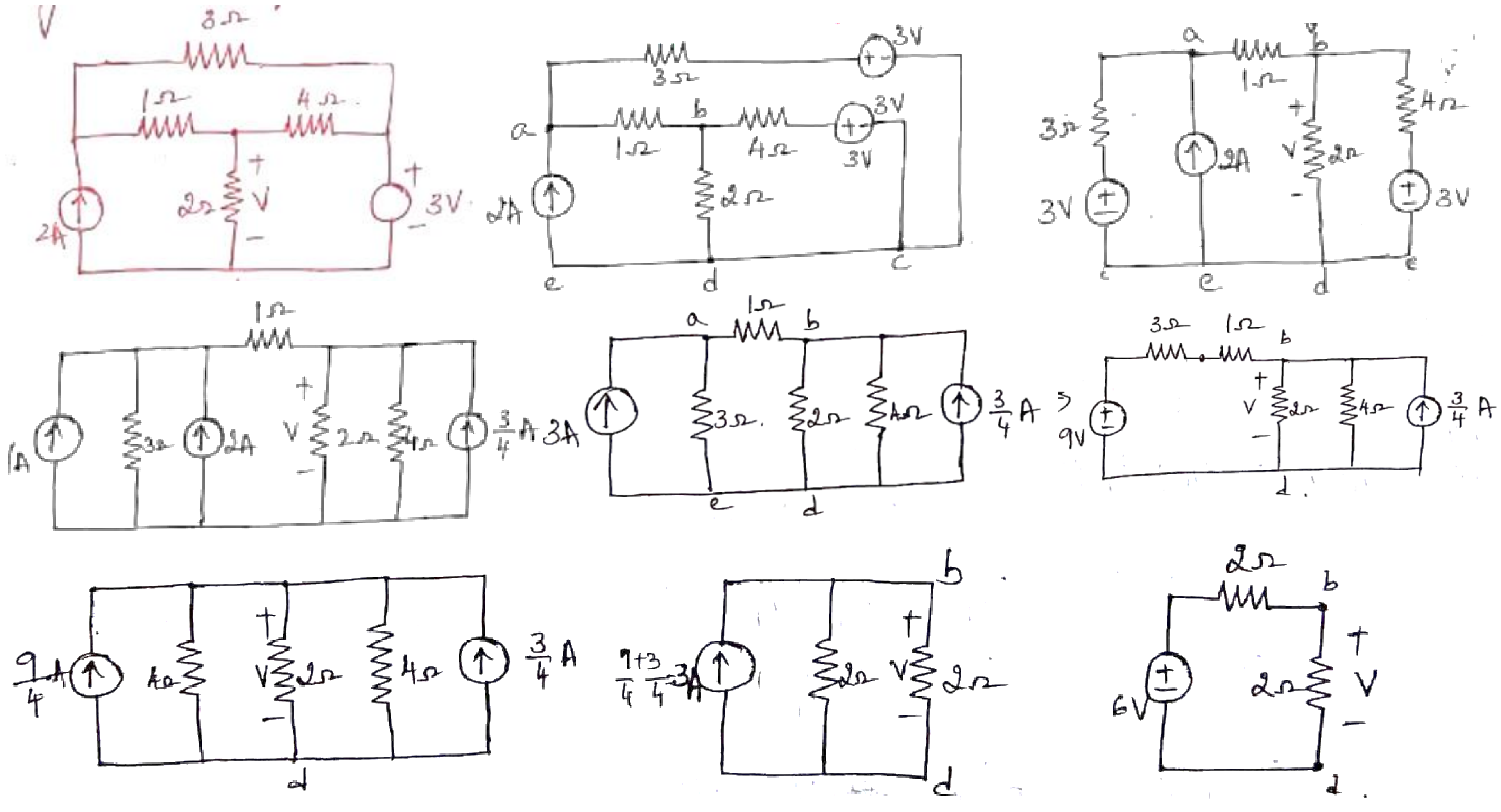






$$I = \frac{2.8}{3.1} = \underline{0.9032A}$$

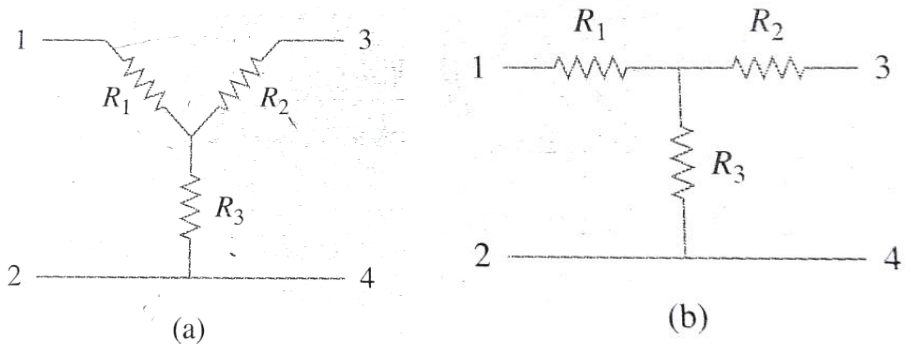
2. For the network shown in fig. Determine the voltage V using source shift & / or source transformation techniques only. Then verify by node equations.



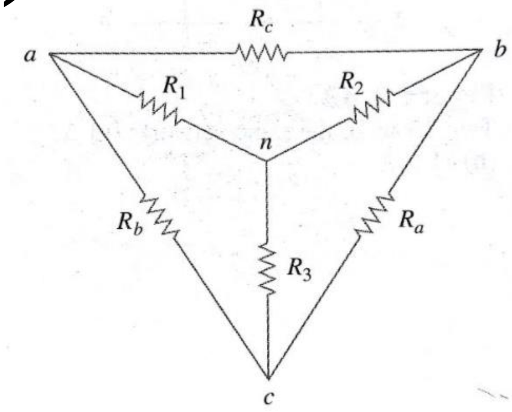
I = 6/4 = 1.5A

V= 2*1.5 = 3V

Delta (Δ) to Wye /Star(Y) Conversion



Two forms of the same network: (a) Y, (b) T.



Superposition of Y and Δ networks as an aid in transforming one to the other.

$R_{12}(Y) = \ R_1 + R_3$ ---------(1)

$$R_{12}(\Delta) = \ R_b||(R_a + R_C)$$

Setting $R_{12}(Y) = R_{12}(\Delta) gives$

$R_{12} = R_1 + R_3 \ = \ \frac{R_b(R_a+R_C)}{R_a+R_b+R_C}$ ---------(2)

Similarly,

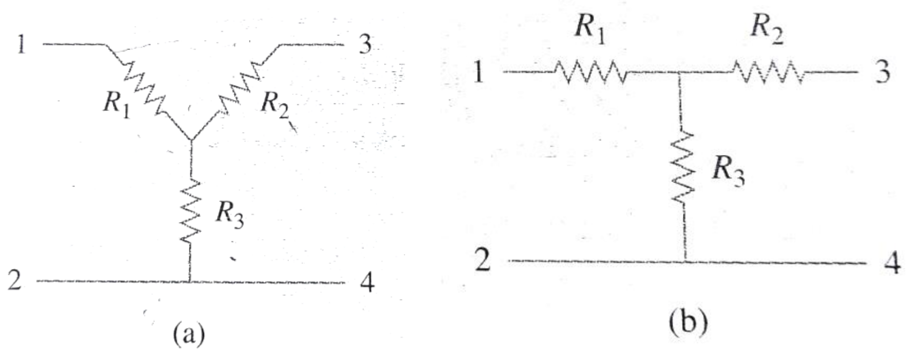
$R_{13} = R_1 + R_2 \ = \ \frac{R_C(R_a+R_b)}{R_a+R_b+R_C}$ ---------(3)

$R_{34} = R_2 + R_3 \ = \ \frac{R_a(R_b+R_c)}{R_a+R_b+R_C}$ ---------(4)

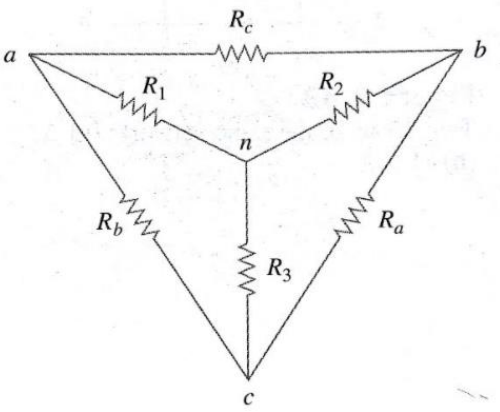
Subtracting equation (2) – (4)

$R_1 - R_2 = \frac{R_b(R_a+R_C)}{R_a+R_b+R_C}$ - $\frac{R_a(R_b+R_c)}{R_a+R_b+R_C} \ = \frac{R_bR_a+R_bR_C-R_aR_b-R_aR_c}{(R_a+R_b+R_C)} = \frac{R_bR_C - R_aR_c}{R_a+R_b+R_C} = \frac{R_C(R_b-R_a)}{R_a+R_b+R_C}$ -----(5)

Delta (Δ) to Wye /Star(Y) Conversion



Two forms of the same network: (a) Y, (b) T.



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$$R_{12}(\Delta) = R_b||(R_a + R_C)$$

Setting $R_{12}(Y) = R_{12}(\Delta) gives$

$R_{12} = R_1 + R_3 = \frac{R_b(R_a+R_C)}{R_a+R_b+R_C}$ ---------(2)

Similarly,

$R_{13} = R_1 + R_2 = \frac{R_C(R_a+R_b)}{R_a+R_b+R_C}$ ---------(3)

$R_{34} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_C}$ ---------(4)

Subtracting equation (2) – (4)

$R_1 - R_2 = \frac{R_b(R_a+R_C)}{R_a+R_b+R_C} - \frac{R_a(R_b+R_c)}{R_a+R_b+R_C} = \frac{R_bR_a+R_bR_C-R_aR_b-R_aR_c}{(R_a+R_b+R_C)} = \frac{R_bR_C - R_aR_c}{R_a+R_b+R_C} = \frac{R_C(R_b-R_a)}{R_a+R_b+R_C}$ -----(5)

Adding Equation (3) & (5)

$$R_1 + R_2 + R_1 - R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_C} + \frac{R_C(R_b-R_a)}{R_a+R_b+R_C}$$

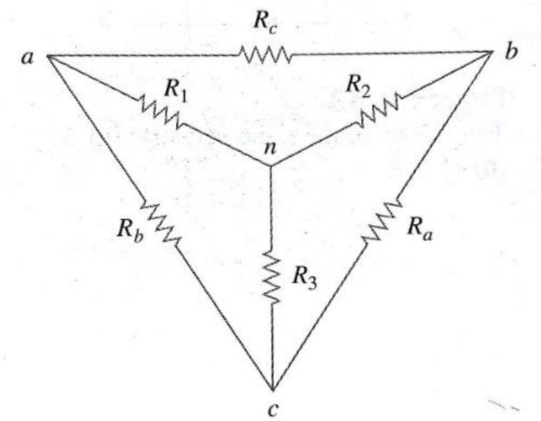
$$2R_1 = \frac{R_c[R_a+R_b+R_b-R_a]}{R_a+R_b+R_C} = \frac{2R_bR_c}{R_a+R_b+R_C}$$

$$R_1 = \frac{R_bR_c}{R_a+R_b+R_C} \text{ -----(6)}$$

Subtracting equation (3) – (5), gives

$$R_2 = \frac{R_cR_a}{R_a+R_b+R_C} \text{ -----(7)}$$

$$R_3 = \frac{R_aR_b}{R_a+R_b+R_C} \text{ -----(8)}$$

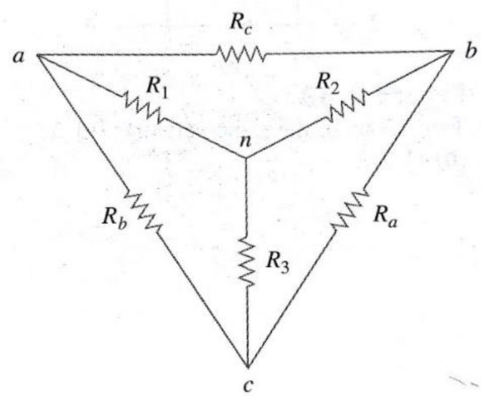


Wye /Star(Y) to Delta (Δ) Conversion

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_C} \text{-----}(6)$$

$$R_2 = \frac{R_c R_a}{R_a+R_b+R_C} \text{-----}(7)$$

$$R_3 = \frac{R_a R_b}{R_a+R_b+R_C} \text{-----}(8)$$



To obtain the conversion formulas for transforming a Y network to an equivalent Δ network, we note from Eqs. (6) to (8) that

$$R_1\ R_2+R_2\ R_3+R_3\ R_1 = \frac{R_a R_b R_c(R_a+R_b+R_C)}{(R_a+R_b+R_C)^2}$$

$$= \frac{R_a R_b R_c}{R_a+R_b+R_C} \quad \text{------}(9)$$

Dividing eq(9) by each of Eqs. (6) to (8) leads to the following equations:

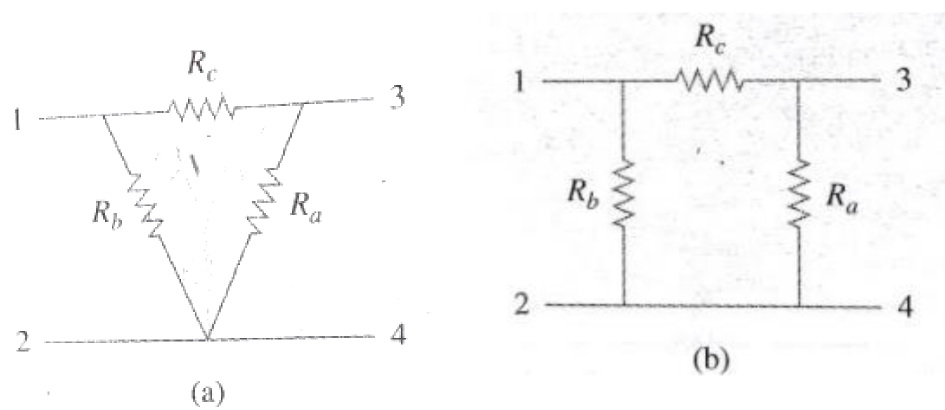
$$R_a = \frac{R_1\ R_2+R_2\ R_3+R_3\ R_1}{R_1} \text{------------}(10)$$

$$R_b = \frac{R_1\ R_2+R_2\ R_3+R_3\ R_1}{R_2} \text{------------}(11)$$

$$R_c = \frac{R_1\ R_2+R_2\ R_3+R_3\ R_1}{R_3} \text{------------}(12)$$

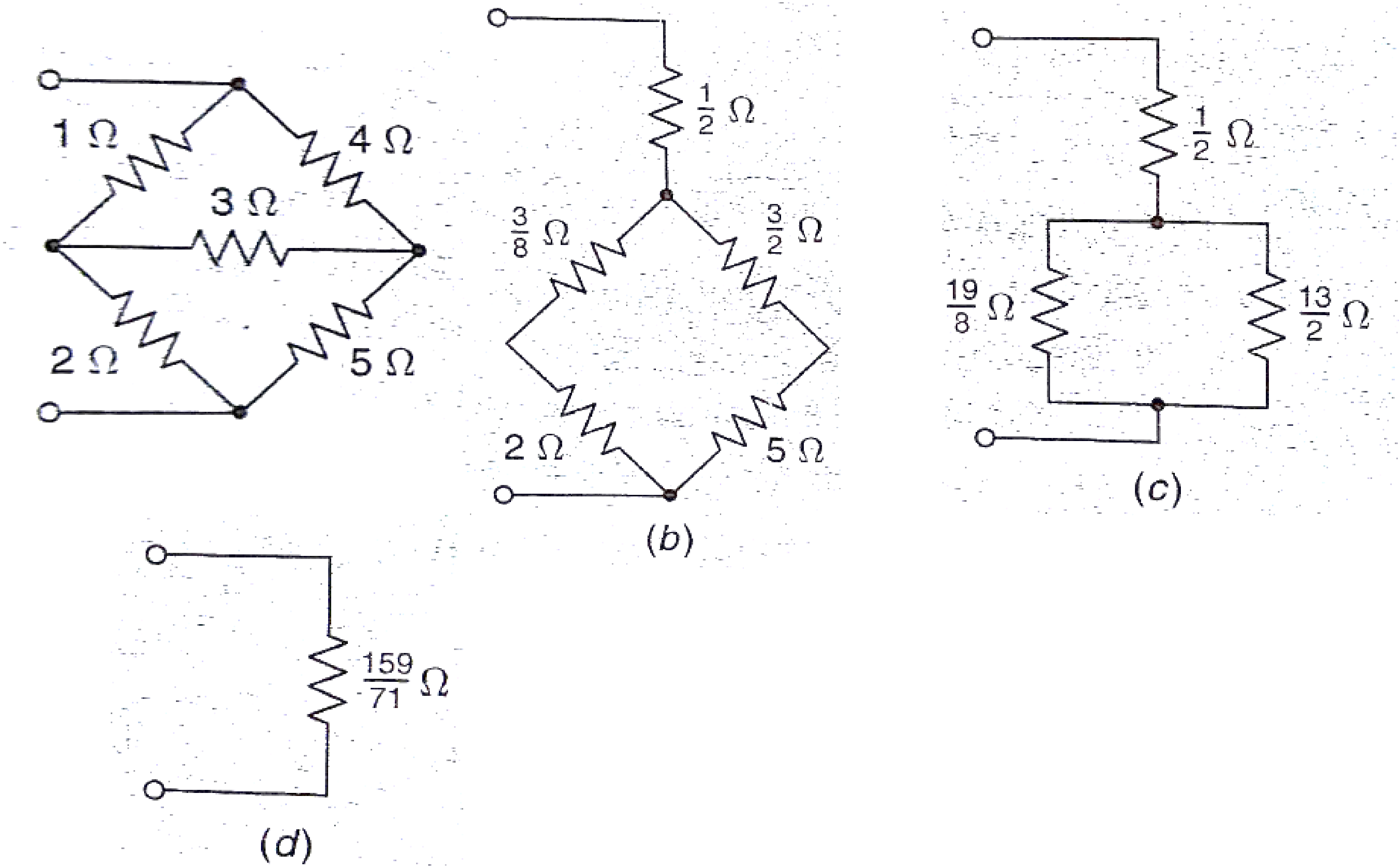
From Eqs. (10) to (12) and fig, the conversion rule for Y to Δ is as follows:

Each resistor in the delta network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

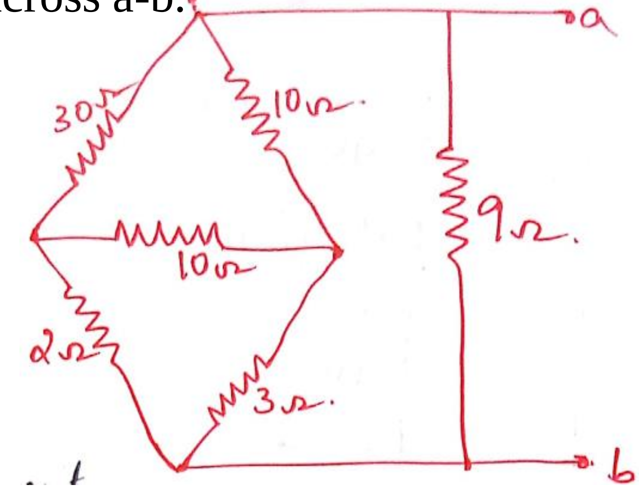


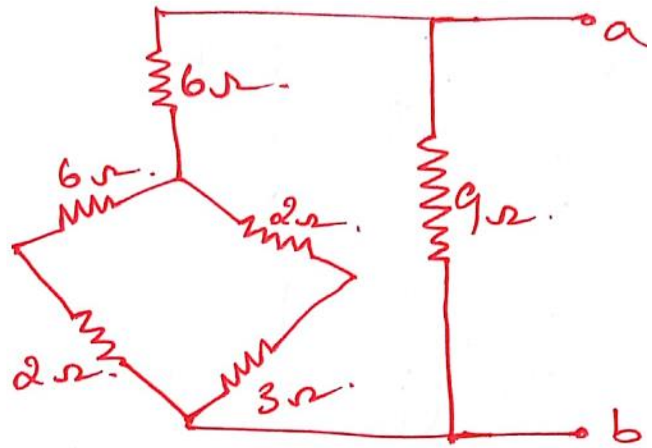
Two forms of the same network: (a) Δ, (b) Π.

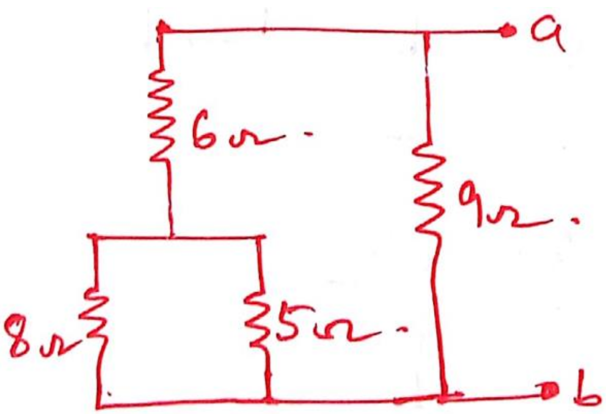
1. Use Star-delta transformation to find the equivalent resistance of the given circuit.

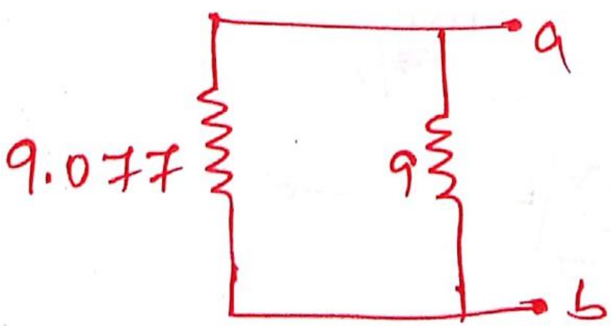


2. Use Star-delta transformation to find the equivalent resistance of the given circuit across a-b.





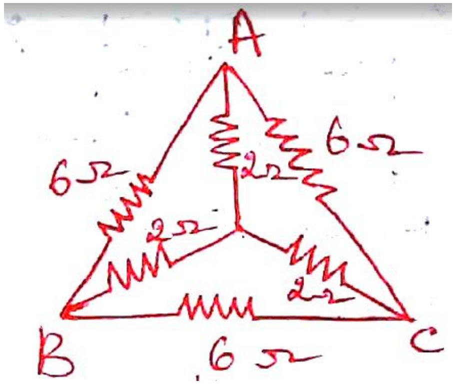




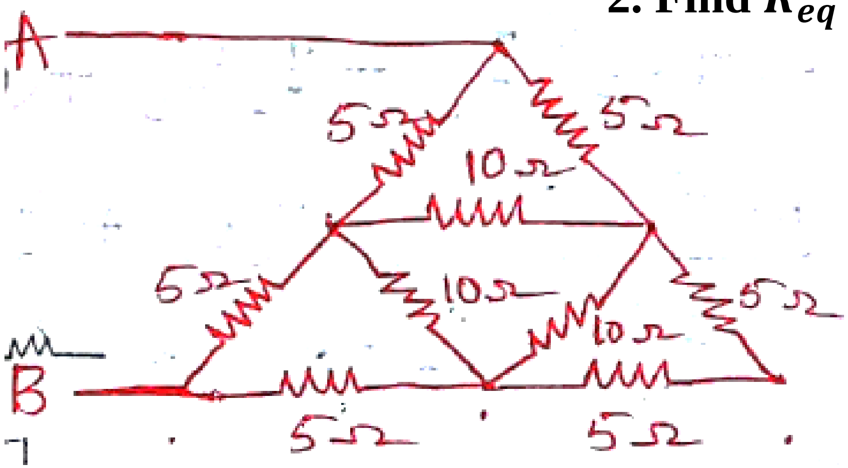
$R_{eq} = 4.52\,\Omega$

Practice Problems

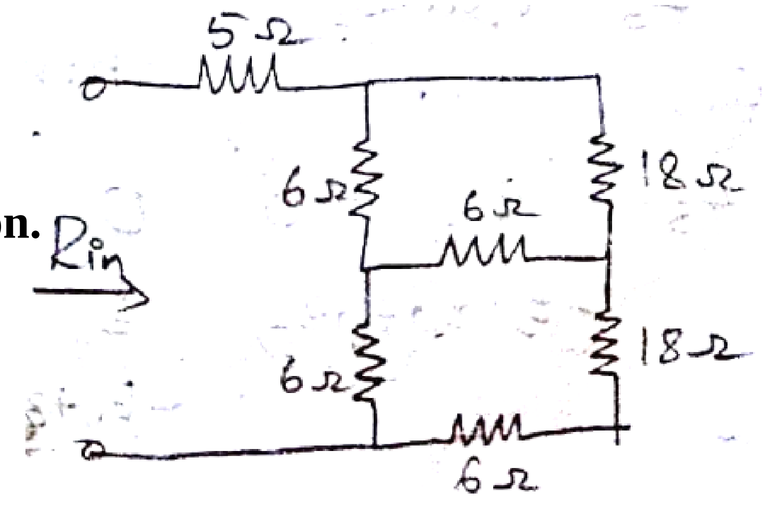
1. **Find R_{eq} using Star – delta transformation.**



2. **Find R_{eq} across A-B using Star – delta transformation.**



3. **Find R_{in} using Star – delta transformation.**



Mesh / Loop Analysis

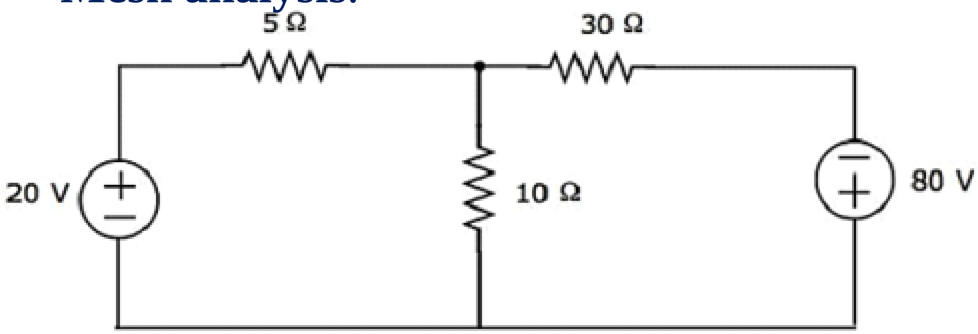
- In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as **Mesh-current method**.
- A **branch** is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.
- If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

Procedure of Mesh Analysis

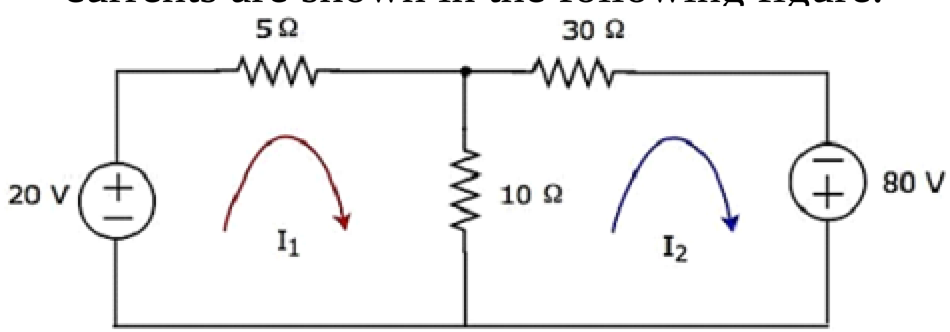
- Follow these steps while solving any electrical network or circuit using Mesh analysis.
- **Step 1** − Identify the **meshes** and label the mesh currents in either clockwise or anti-clockwise direction.
- **Step 2** − Observe the amount of current that flows through each element in terms of mesh currents.
- **Step 3** − Write **mesh equations** to all meshes. Mesh equation is obtained by applying KVL first and then Ohm's law.
- **Step 4** − Solve the mesh equations obtained in Step 3 in order to get the **mesh currents**.

Example

1. Find the voltage across 30 Ω resistor using **Mesh analysis**.



Step 1 − There are two meshes in the above circuit. The **mesh currents** I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.



Step 4 − Finding mesh currents I_1 and I_2 by solving Equation 1 and Equation 2.

$$I_1 = \frac{16}{5} \qquad I_2 = \frac{14}{5}$$

Step 2 − The mesh current I_1 flows through 20 V voltage source and 5 Ω resistor. Similarly, the mesh current I_2 flows through 30 Ω resistor and -80 V voltage source. But, the difference of two mesh currents, I_1 and I_2, flows through 10 Ω resistor, since it is the common branch of two meshes.

Step 3 − In this case, we will get **two mesh equations** since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit.

$$20 - 5I_1 + 10(I_2 - I_1) = 0$$

$$-15I_1 + 10I_2 = -20 \quad ---(1)$$

$$-30I_2 + 80 + 10(I_1 - I_2) = 0$$

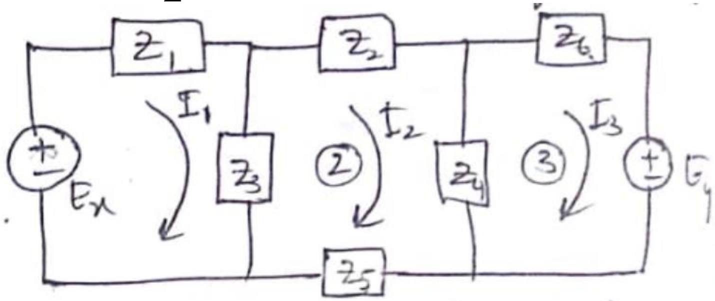
$$10I_1 - 40I_2 = -80 \text{ -----(2)}$$

Step 5 − The current flowing through 30 Ω resistor is nothing but the mesh current I_2

Now, we can find the voltage across 30 Ω resistor by using Ohm's law.

$$V_{30\Omega} = I_2 R \qquad V_{30\Omega} = \left(\frac{14}{5}\right) * 30 = 84\text{V}$$

Example to understand the Cramer's Rule.



In mesh (1), $E_x - I_1 z_1 + (I_2 - I_1) z_3 = 0$.

$$-(z_1+z_3) I_1 + I_2 z_3 = -E_x$$

$$-(z_1+z_3) I_1 + I_2 z_3 + (0) I_3 = -E_x \rightarrow (1)$$

In mesh (2), $-I_2 z_2 + (I_3 - I_2) z_4 - I_2 z_5 + (I_1 - I_2) z_3 = 0$

$$I_1 z_3 - (z_2+z_3+z_4+z_5) I_2 + z_4 I_3 = 0 \rightarrow (2)$$

In mesh (3); $-z_6 I_3 - E_y + (I_2 - I_3) z_4 = 0$.

$$(0) I_1 + z_4 I_2 - (z_4+z_6) I_3 = E_y \rightarrow (3)$$

$$\begin{bmatrix} -(z_1+z_3) & z_3 & 0 \\ z_3 & -(z_2+z_3+z_4+z_5) & z_4 \\ 0 & z_4 & -(z_4+z_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -E_x \\ 0 \\ +E_y \end{bmatrix}$$

By Cramer's rule

$$I_1 = \frac{1}{\Delta} \begin{vmatrix} -E_x & z_3 & 0 \\ 0 & -(z_2+z_3+z_4+z_5) & z_4 \\ E_y & z_4 & -(z_4+z_6) \end{vmatrix}$$

$$I_2 = \frac{1}{\Delta} \begin{vmatrix} -(z_1+z_3) & -E_x & 0 \\ z_3 & 0 & z_4 \\ 0 & E_y & -(z_4+z_6) \end{vmatrix}$$

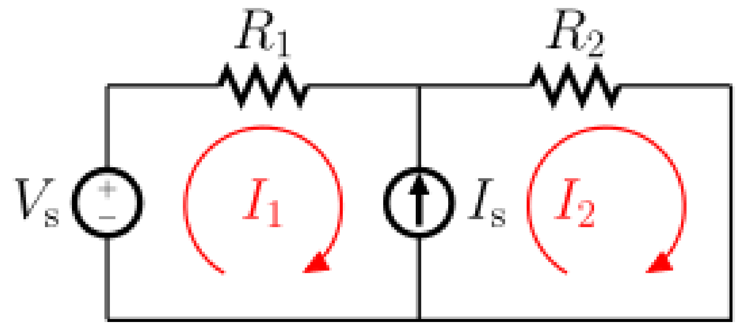
$$\text{Where } \Delta = \begin{vmatrix} -(z_1+z_3) & z_3 & 0 \\ z_3 & -(z_2+z_3+z_4+z_5) & z_4 \\ 0 & z_4 & -(z_4+z_6) \end{vmatrix}$$

$$I_3 = \frac{1}{\Delta} \begin{vmatrix} -(z_1+z_3) & z_3 & -E_x \\ z_3 & -(z_2+z_3+z_4+z_5) & 0 \\ 0 & z_4 & E_y \end{vmatrix}$$

Note:

1. While assuming loop currents make sure that atleast one loop current links with every element.
2. No two loops should be identical
3. Choose minimum number of loop currents.
4. Convert current sources if present, into their equivalent voltage sources for loop analysis, whenever possible.
5. If current in a particular branch is required, then try to choose loop current in such a way that only one loop current links with that branch.

Concept of Super Mesh

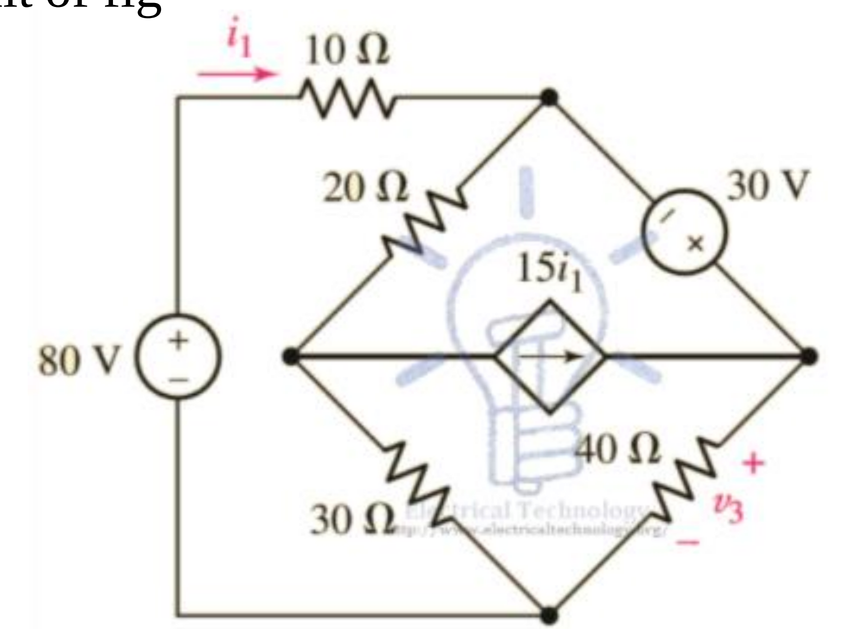


- A supermesh occurs when a current source is contained between two essential meshes.
- The following is a simple example of dealing with a supermesh.

Current source: $I_s = I_2 - I_1$

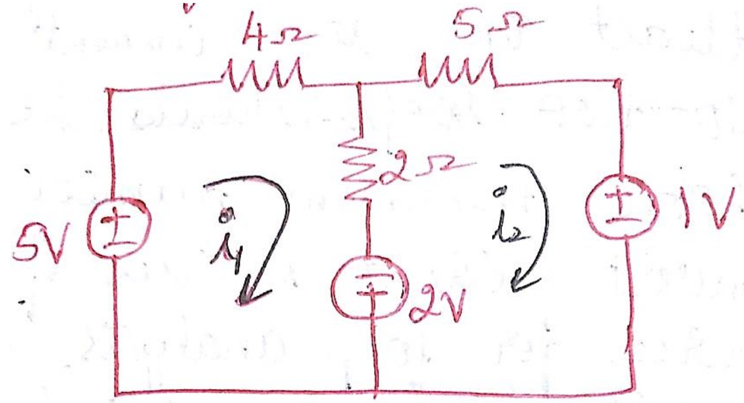
Mesh 1, 2: $-V_s + R_1 I_1 + R_2 I_2 = 0$

Ex. Determine V_3 by Supermesh in the circuit of fig below

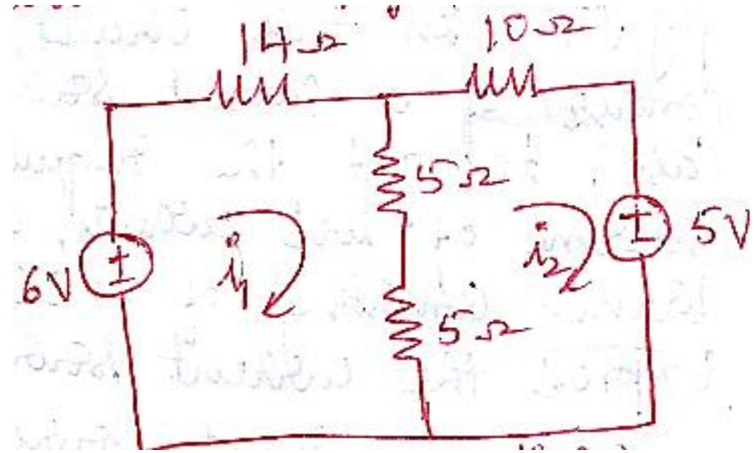


V3 = 104.2 V, i_1 = 0.583 A, i_2 = -6.15 A, i_3 = 2.6 A

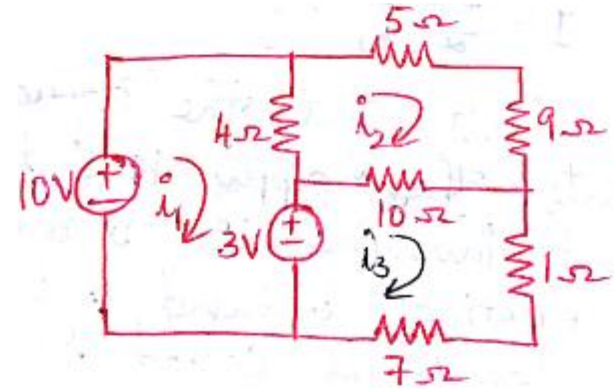
1. Determine the power supplied by the 2V source of the fig shown.



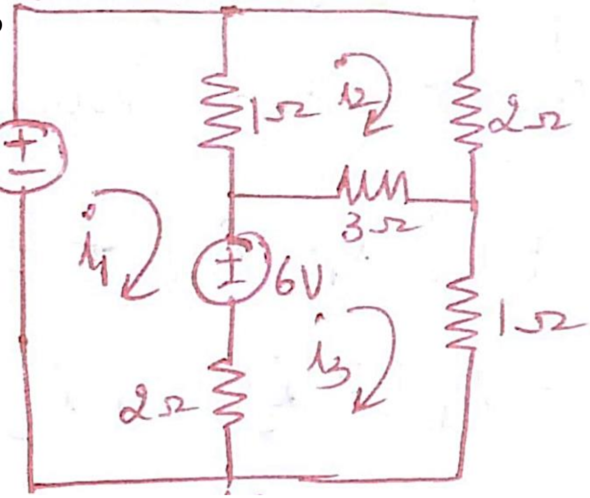
2. Determine i_1 & i_2 in the circuit shown in fig



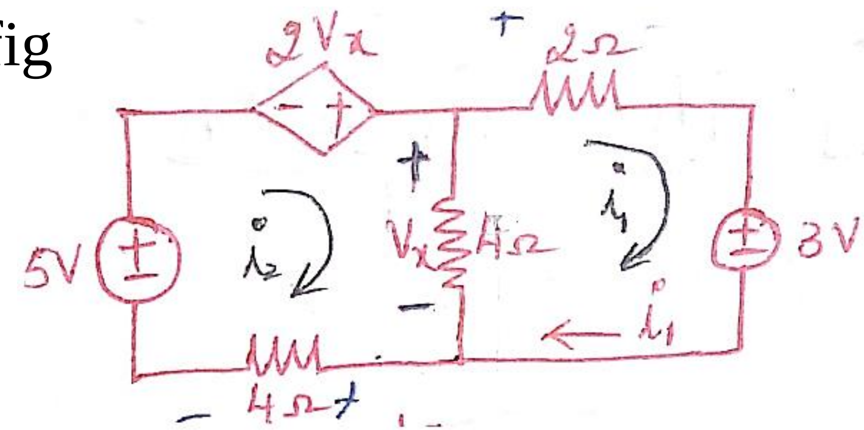
3. Determine i_1 , i_2 & i_3 in the circuit shown in fig



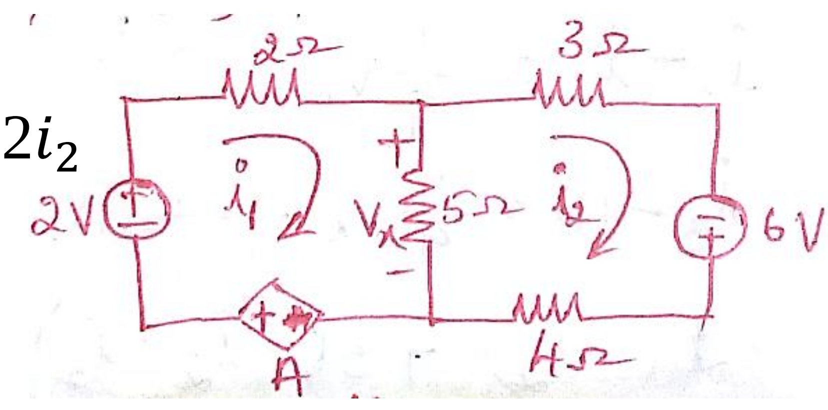
4. Determine i_1 , i_2 & i_3 in the circuit shown in fig



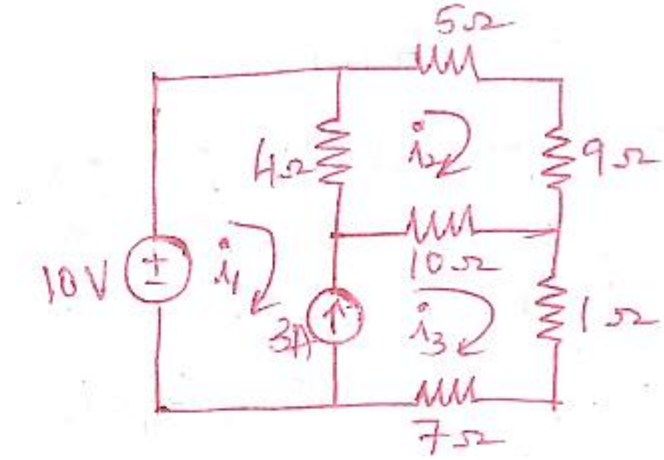
5. Determine i_1 in the circuit shown in fig



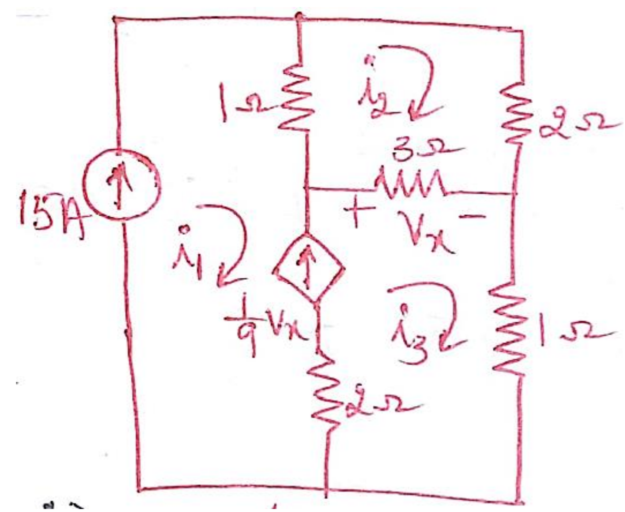
6. Determine i_1 in the circuit shown in fig
If the controlling quantity A is equal to a) $2i_2$
b) $2v_x$

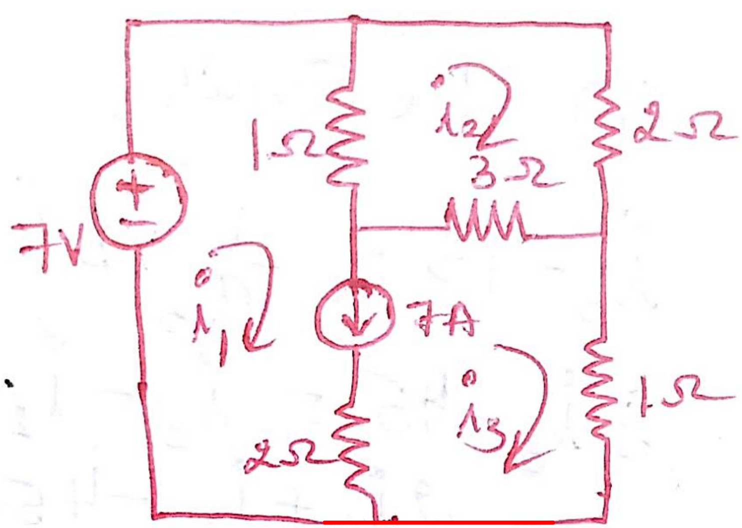


7. Determine i_1 in the circuit shown in fig



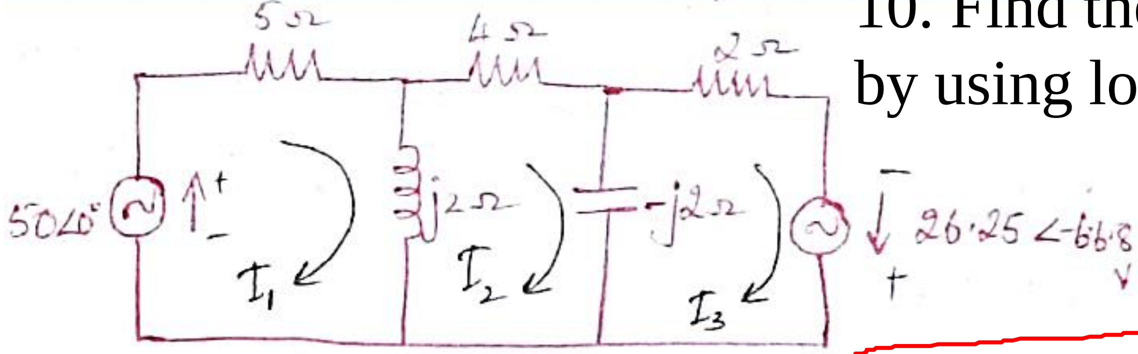
8. Use mesh analysis to evaluate the three unknown currents in the circuit of fig





9. Use mesh analysis to evaluate the three unknown currents in the circuit of fig

10. Find the current through 4Ω resistor by using loop method.



Loop 1,

$$50\angle 0^\circ - 5I_1 + j2(I_2 - I_1) = 0$$

$$(-5-j2)I_1 + j2I_2 = -50\angle 0^\circ$$

$$(5+j2)I_1 - j2I_2 = 50\angle 0^\circ \longrightarrow (1)$$

At loop 2,

$$j2(I_1 - I_2) - 4I_2 - j2(I_3 - I_2) = 0$$

$$j2I_1 - 4I_2 - j2I_3 = 0 \longrightarrow (2)$$

At loop 3,

$$26.25\angle -66.8 - j2(I_2 - I_3) - 2I_3 = 0.$$

$$-j2I_2 + (-2+j2)I_3 = -26.25\angle -66.8$$

$$j2I_2 + (2-j2)I_3 = 26.25\angle -66.8 \longrightarrow (3)$$

$$\Delta = \begin{vmatrix} (5+j2) & -j2 & 0 \\ j2 & -4 & -j2 \\ 0 & j2 & (2-j2) \end{vmatrix}$$

$$= (5+j2)[-4\times(2-j2) + j2\times j2] + j2[j2(2-j2)]$$

$$= -76 + j16 - 8 + j8 \quad = \underline{\underline{-84+j24}}$$

$$\Delta_2 = \begin{vmatrix} 5+j2 & 50\angle 0^\circ & 0 \\ j2 & 0 & -j2 \\ 0 & 26.25\angle -66.8 & 2-j2 \end{vmatrix}$$

$$= (5+j2)[j2\times 26.25\angle -66.8] - 50\angle 0^\circ [j2(2-j2)]$$

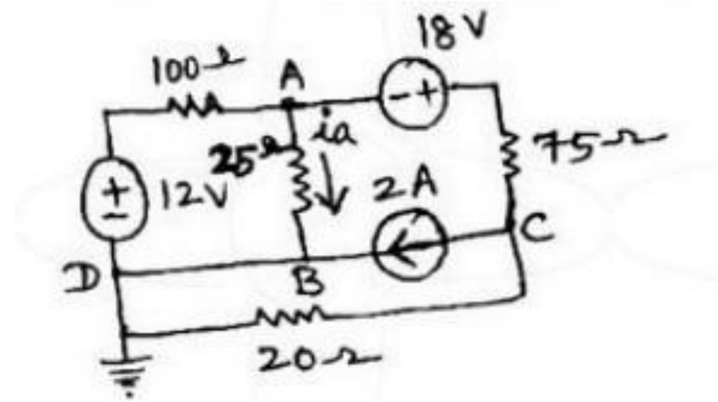
$$= 199.9 + j199.9 - 200 - j200$$

$$\simeq \underline{\underline{0}}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \underline{\underline{0\,A}}$$

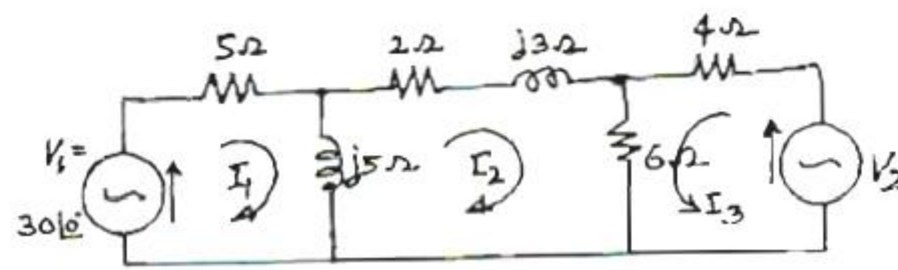
VTU Question Paper questions.

1. Find the current i_a in the circuit given in fig using mesh analysis.

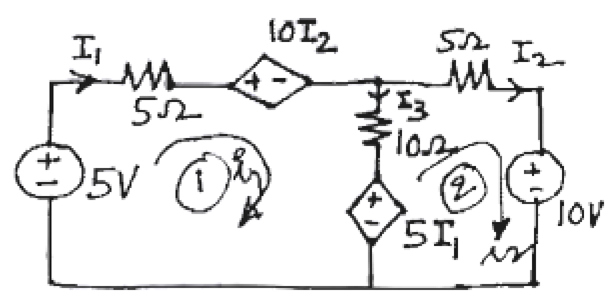


2.. Determine i_1 , i_2 & i_3 in the circuit shown in fig

3. In the network of fig determine v_2 such that the current in the impedance (2+j3) is zero. Use Mesh analysis.

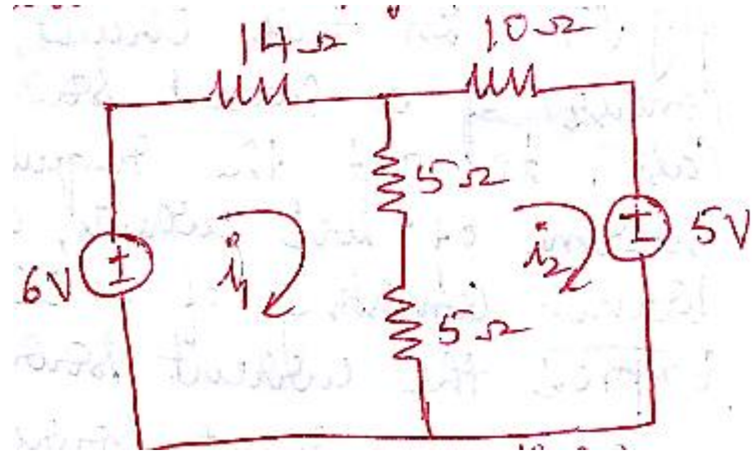


4. Use mesh analysis to determine the branch currents in the network shown in fig.

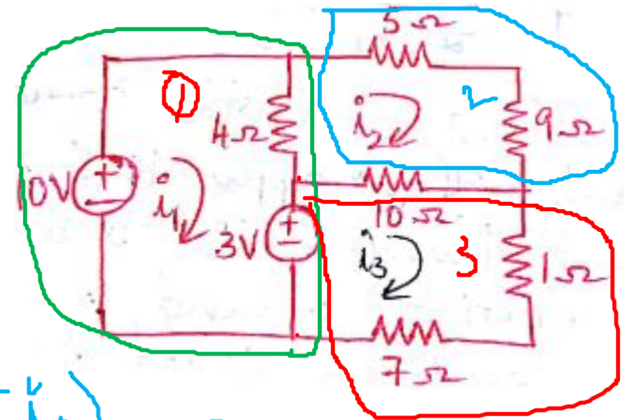


2. Determine i_1 & i_2 in the circuit shown in fig

$i_1 = 0.18\,A$ $\quad$ $i_2 = -0.15\,A$



3. Determine i_1 , i_2 & i_3 in the circuit shown in fig



$10 + 4(-i_1 + i_2) - 3 = 0$

$a_1 \quad b_1 \quad c_1 \quad d_1$

$-4i_1 + 4i_2 + 0 = -7 \longrightarrow (1)$

$-5i_2 - 9i_2 + 10(-i_2 + i_3) + 4(-i_2 + i_1) = 0$

$4i_1 - 28i_2 + 10i_3 = 0 \longrightarrow (2)$

$a_2 \quad b_2 \quad c_2 \quad d_2$

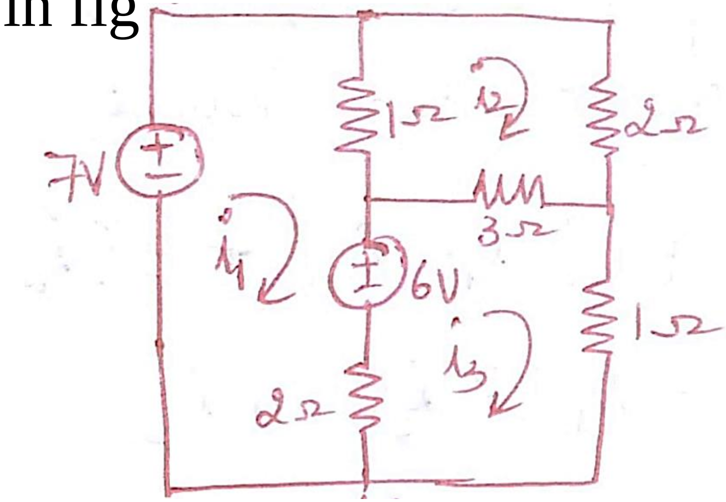
$10(-i_3 + i_2) + (-1\, i_3) - 7i_3 + 3 = 0$

$0i_1 + 10i_2 - 18i_3 = -3 \longrightarrow (3)$

$a_3 \quad b_3 \quad c_3 \quad d_4$

$i_1 = 2.24A \quad i_2 = 0.47A \quad i_3 = 0.43A$

4. Determine i_1 , i_2 & i_3 in the circuit shown in fig



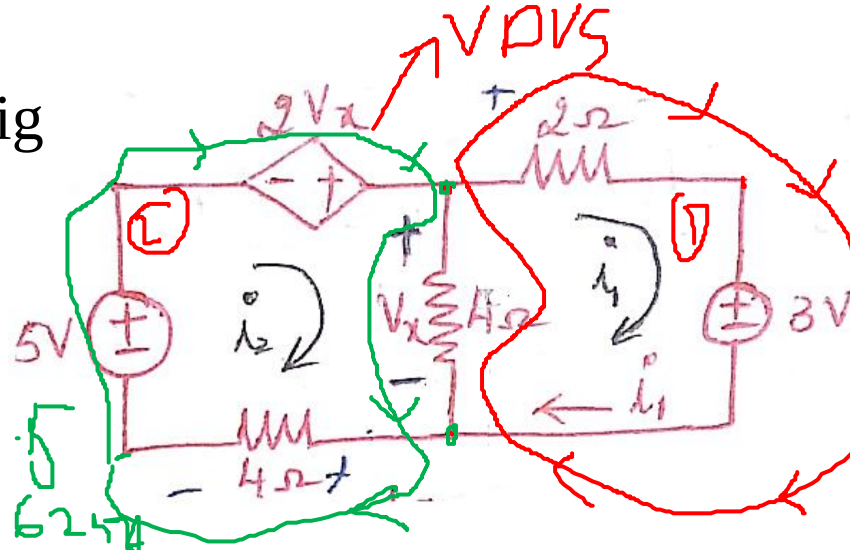
A:

$i_1 = 3A$

$i_2 = 2A$

$i_3 = 3A$

5. Determine i_1 in the circuit shown in fig



$$-2i_1 - 3 + 4(-i_1 + i_2) = 0$$

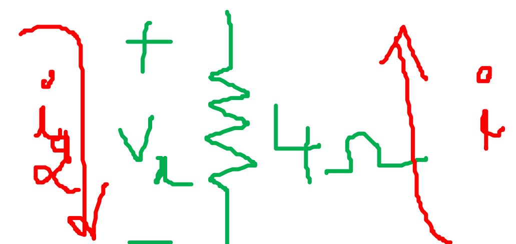
$$-5\times 1.25 + 4i_2 = 3 \; ; \quad 4i_2 = 10.5$$

$$i_2 = 2.625$$

$$-6i_1 + 4i_2 = 3 \rightarrow (1)$$

$$5 + 2V_x + 4(-i_2 + i_1) - 4i_2 = 0$$

$$4i_1 - 8i_2 + 2V_x = -5 \longrightarrow (2)$$



$$\boxed{V_x = (i_2 - i_1)4}$$

$$4i_1 - 8i_2 + 2(i_2 - i_1)4 = -5$$

$$V = iR$$

$$(i_2 - i_1) = ?$$

$$(i_1 - i_2) = ?$$

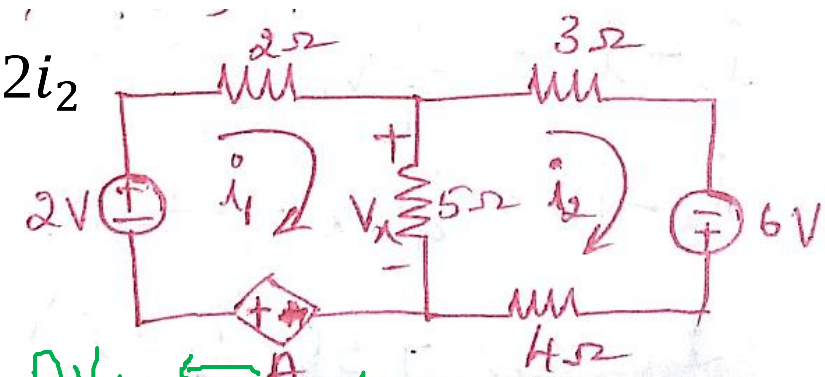
$$4i_1 - \cancel{8i_2} + \cancel{8i_2} - 8i_1 = -5$$

$$-4i_1 = -5$$

$$i_1 = 5/4\,A$$

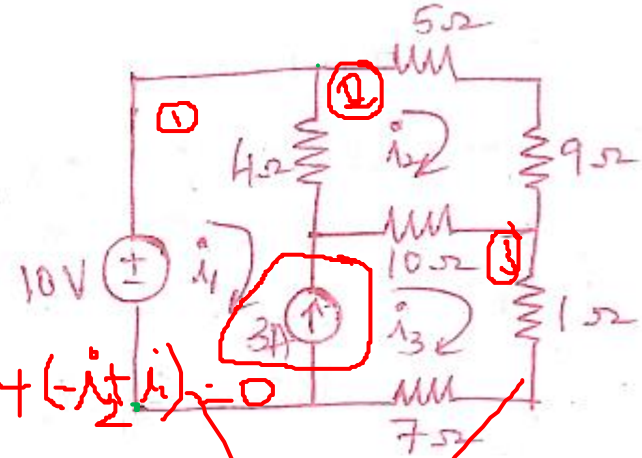
6. Determine i_1 in the circuit shown in fig
If the controlling quantity A is equal to a) $2i_2$
b) $2v_x$

$i_1 =$



VDVS ← $2V_x$ ← A → $2i_2$ → CDVS

7. Determine i_1 in the circuit shown in fig



$-i_1 + 0i_2 + i_3 = 3 \rightarrow (A)$

$i_3 - i_1 = 3 \rightarrow (1)$

Loop 2: $-5i_2 - 9i_2 + 10(-i_2 + i_3) + 4(-i_2 + i_1) = 0$

$4i_1 - 28i_2 + 10i_3 = 0 \rightarrow (A)$

Supermesh

$10 - 5i_2 - 9i_2 - 1i_3 - 7i_3 = 0$

$0i_1 - 14i_2 - 8i_3 = -10 \rightarrow (B)$

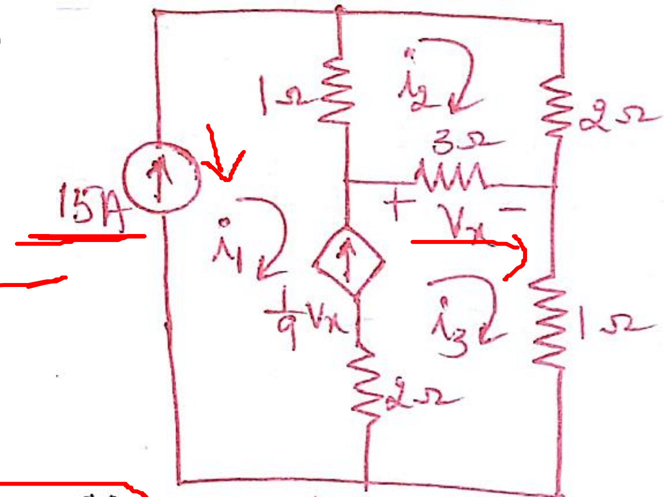
$i_1 = -1.43A$

$i_2 = 0.104A$

$i_3 = 1.066A$

#8.

8. Use mesh analysis to evaluate the three unknown currents in the circuit of fig



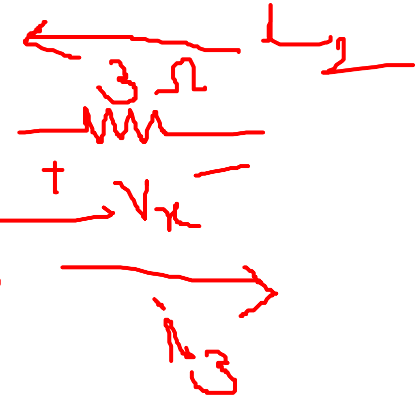
$i_3 - i_1 = \frac{1}{9}V_x$

$i_1 = 15A$

$i_3 - i_1 = \frac{1}{\cancel{9}_{3}}(i_3 - i_2)\cancel{3}$

$i_3 - 15 = \frac{i_3}{3} - \frac{i_2}{3}$

$\boxed{\begin{array}{l} i_2 = 11A \\ i_3 = 17A \end{array}}$

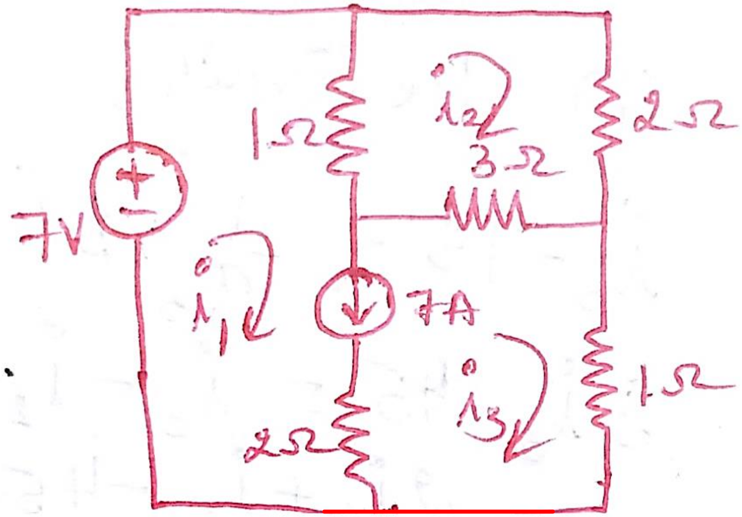


$\frac{i_2}{3} - 0.67\, i_3 = 15 \rightarrow (1)$

$V_x = (i_3 - i_2)3$

$1(-i_2 + i_1) - 2i_2 + 3(-i_2 + i_3) = 0$

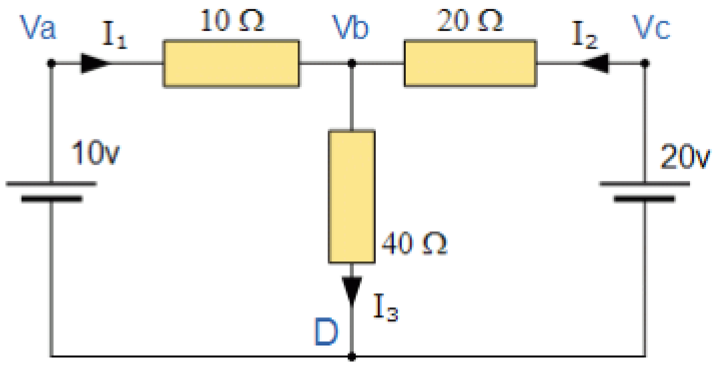
$-6i_2 + 3i_3 = -15 \rightarrow (2)$



9. Use mesh analysis to evaluate the three unknown currents in the circuit of fig

Nodal Voltage Analysis

Nodal Voltage Analysis finds the unknown voltage drops around a circuit between different nodes that provide a common connection for two or more circuit components



In the above circuit, node D is chosen as the reference node and the other three nodes are assumed to have voltages, Va, Vb and Vc with respect to node D

V_a = 10V; $V_c = 20V$

$$\frac{10-V_b}{10} + \frac{20-V_b}{20} - \frac{V_b-0}{40} = 0$$

$$\left(1-\frac{Vb}{10}\right)+\left(1-\frac{Vb}{20}\right)=\frac{Vb}{40}$$

$$I_1 + I_2 - I_3 = 0$$

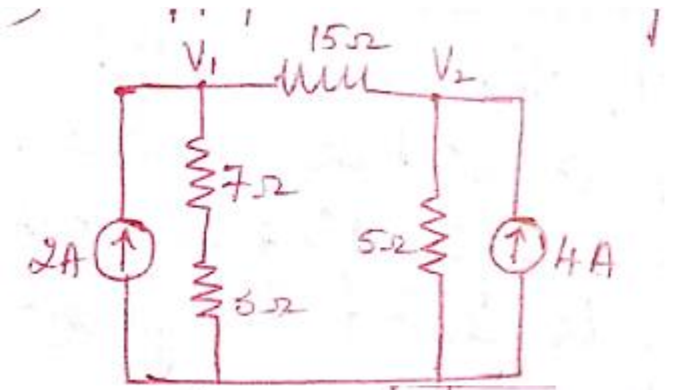
$$2 = Vb\left(\frac{1}{40}+\frac{1}{20}+\frac{1}{10}\right)$$

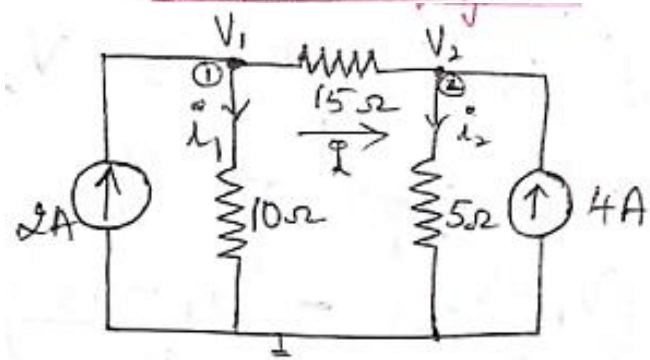
$$Vb = \frac{80}{7}V$$

$$\frac{V_a-V_b}{10} + \frac{V_c-V_b}{20} - \frac{V_b-0}{40} = 0$$

$$\therefore I_3 = \frac{2}{7} \ or \ 0.286 Amps$$

1. Determine the current flowing left to right through the 15Ω resistor of fig shown. Apply Nodal analysis.





Apply KCL at node ①:

$2 - i_1 - i = 0$.

$2 - \frac{V_1}{10} - \left(\frac{V_1 - V_2}{15}\right) = 0$

$-\frac{V_1}{10} - \frac{V_1}{15} + \frac{V_2}{15} + 2 = 0$

$\frac{V_1}{6} - \frac{V_2}{15} = 2 \longrightarrow (1)$

Apply KCL at node ②.

$i - i_2 + 4 = 0$.

$\frac{V_1 - V_2}{15} - \frac{V_2}{5} + 4 = 0$.

$\boxed{I_{15\Omega} = \frac{V_1 - V_2}{15} = \frac{20-20}{15} = 0}$

$\frac{V_1}{15} - \frac{V_1}{15} - \frac{1}{15}V_2 - \frac{V_2}{5} + 4 = 0$

$\frac{V_1}{15} + \frac{2}{15}V_2 = 4 \longrightarrow (2)$

$\frac{V_1}{15} - \frac{4}{15}V_2 = -4 \longrightarrow (2)$

Solving (1) & (2) $V_1 = 20V$; $V_2 = 20V$

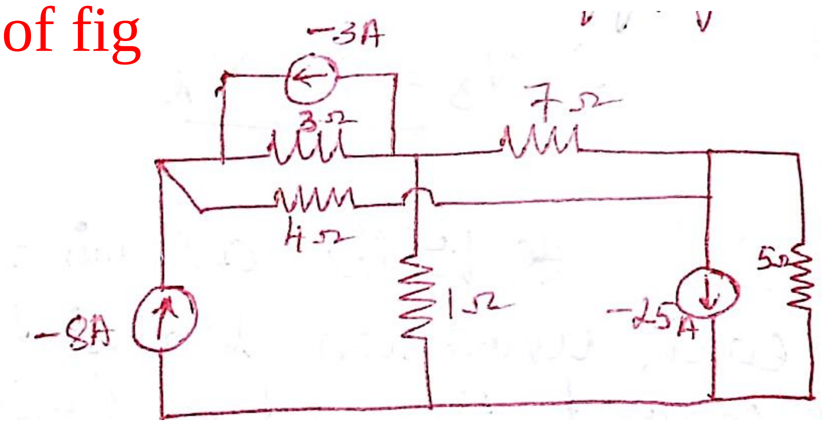
$V_1 = 20V$

$V_2 = 20V$

Current through 15Ω resistor is

$i = \frac{V_1 - V_2}{15} = 0.$

2. Find the node voltages in the circuit of fig shown.



At node V_1,

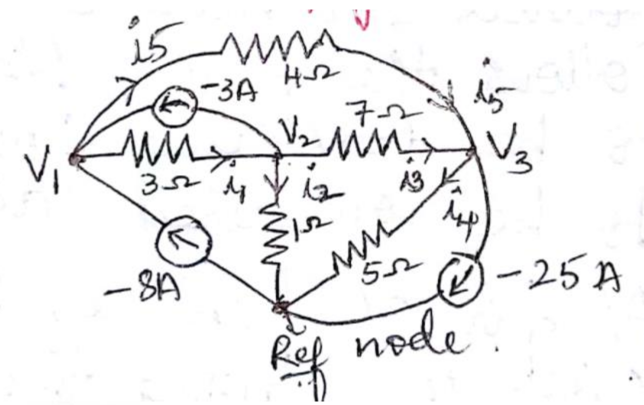
$$-i_1 + (-3) + (-8) - i_5 = 0$$

$$-11 - \left(\frac{V_1 - V_2}{3}\right) - \left(\frac{V_1 - V_3}{4}\right) = 0.$$

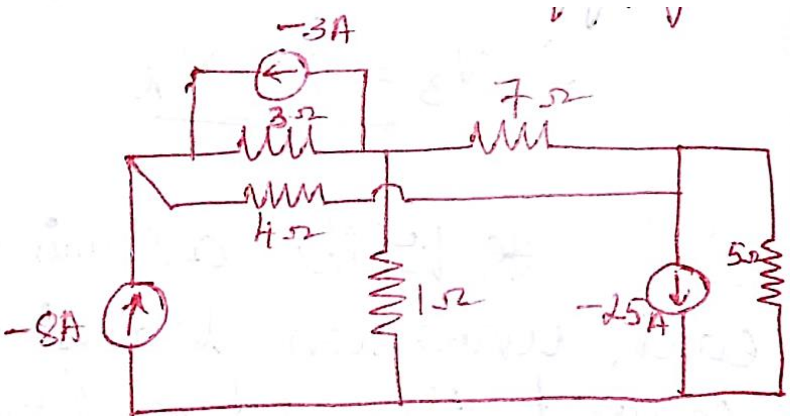
$$-\frac{V_1}{3} - \frac{V_1}{4} + \frac{V_2}{3} + \frac{V_3}{4} = 11$$

$$-\frac{7}{12} V_1 + \frac{V_2}{3} + \frac{V_3}{4} = 11$$

$$-0.5833V_1 + 0.333V_2 + 0.25V_3 = 11 \longrightarrow (1)$$



At V_2; $i_1 - i_2 - (-3) - i_3$



$$\left(\frac{V_1 - V_2}{3}\right) - \frac{V_2}{1} + 3 - \left(\frac{V_2 - V_3}{7}\right) = 0$$

$$+\frac{V_1}{3} - \frac{31}{21} V_2 + \frac{1}{7} V_3 = -3$$

$$0.333 V_1 - 1.476 V_2 + 0.1428 V_3 = -3 \longrightarrow (2)$$

At V_3; $i_3 - i_4 + i_5 - (-25) = 0$

$$\left(\frac{V_2 - V_3}{7}\right) - \frac{V_3}{5} + \left(\frac{V_1 - V_3}{4}\right) + 25 = 0.$$

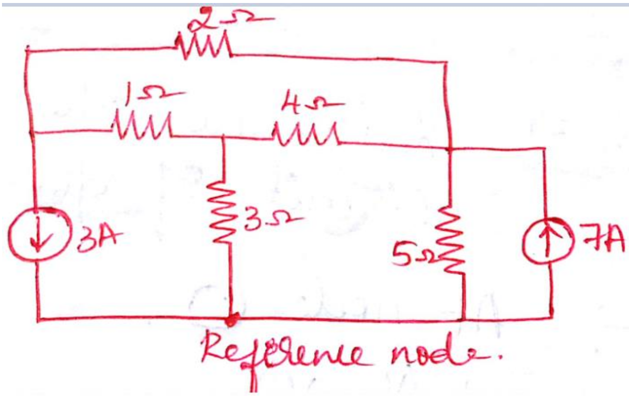
$$\frac{V_1}{4} + \frac{V_2}{7} - \frac{3}{140} V_3 = -25.$$

$$0.25 V_1 + 0.1428 V_2 - 0.5928 V_3 = -25 \longrightarrow (3)$$

Solving equation (1), (2) & (3) we get

$V_1 = 5.41V$; $V_2 = 7.734V$; $V_3 = 46.32V$

3. For the Circuit shown in fig, Compute the voltage across each current source. Apply nodal analysis.



Procedure for solving the Super nodal Circuit.

Step1: Identify the node and reference node.

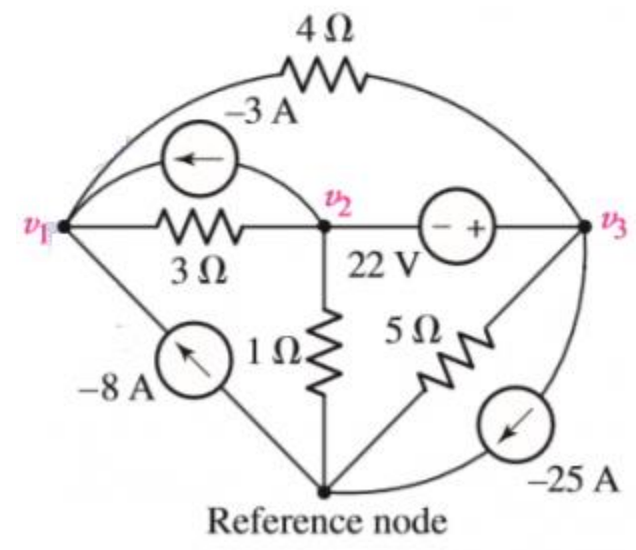
Step2: Check for any nodes connected to voltage source.
If Yes then Those nodes are Super nodes.

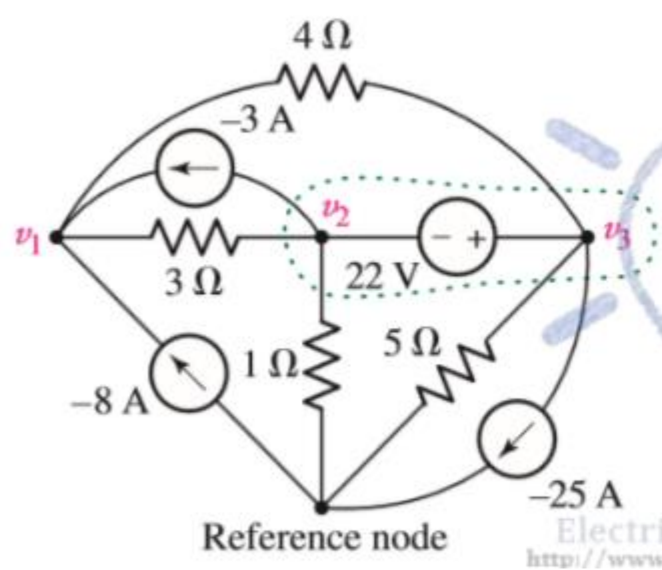
Step 3: Establish the equations for supernodes for the branch containing voltage source.

Step 4: Build the equations for remaining nodes by applying KCL.

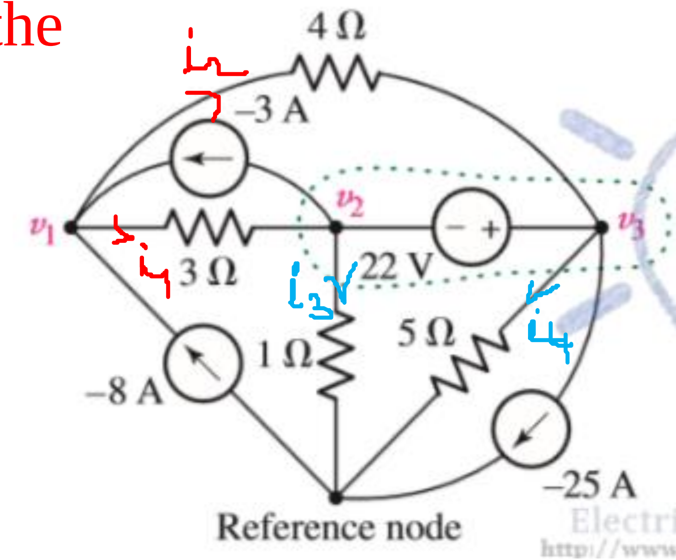
Step 5: Also arrive the equation jointly for the 2 supernodes connecting to the voltage source by excluding the branch having voltage source.

Step 6: Solve the simultaneous equations to obtain the node voltages.





1. Find the node voltage V_1, V_2 and V_3 in the circuit diagram shown in fig using nodal analysis



$$V_3 - V_2 = 22$$

$$0V_1 - V_2 + V_3 = 22 \rightarrow (1)$$

At node 1, KCL,

$$+(-8) - i_1 + (-3) - i_2 = 0$$

$$-8 - \left(\frac{V_1 - V_2}{3}\right) - 3 - \left(\frac{V_1 - V_3}{4}\right) = 0$$

$$-\left(\frac{1}{3} + \frac{1}{4}\right)V_1 + \frac{V_2}{3} + \frac{V_3}{4} = 11 \rightarrow (2)$$

KCL to node 2 & 3

$$i_1 - (-3) - i_3 - i_4 - (-25) + i_2 = 0$$

$$\left(\frac{V_1 - V_2}{3}\right) + 3 - \left(\frac{V_2 - 0}{1}\right) - \left(\frac{V_3 - 0}{5}\right) + 25 + \left(\frac{V_1 - V_3}{4}\right) = 0$$

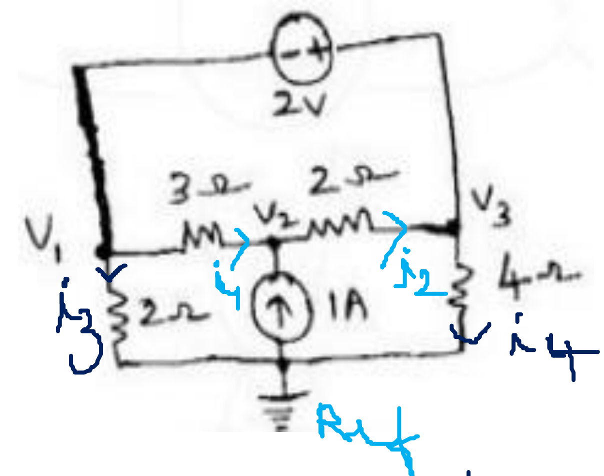
$$\left(\frac{1}{3} + \frac{1}{4}\right)V_1 - \left(\frac{1}{3} + 1\right)V_2 - \left(\frac{1}{5} + \frac{1}{4}\right)V_3 = -28 \rightarrow (3)$$

$V_1 = 1.071$ V

$V_2 =$ V

$V_3 =$ V

2. Find the node voltage V_1, V_2 and V_3 in the circuit diagram shown in fig using nodal analysis



$V_3 - V_1 = 2$

$-V_1 + 0V_2 + V_3 = 2 \rightarrow (1)$

KCL to node, $i_1 + 1 - i_2 = 0$

$\left(\frac{V_1 - V_2}{3}\right) + 1 - \left(\frac{V_2 - V_3}{2}\right) = 0$

$\frac{V_1}{3} - \left(\frac{1}{3} + \frac{1}{2}\right)V_2 + \frac{V_3}{2} = -1 \rightarrow (2)$

$-i_1 - i_3 + i_2 - i_4 = 0$

$-\left(\frac{V_1 - V_2}{3}\right) - \left(\frac{V_1 - 0}{2}\right) + \left(\frac{V_2 - V_3}{2}\right) - \left(\frac{V_3 - 0}{4}\right) = 0$

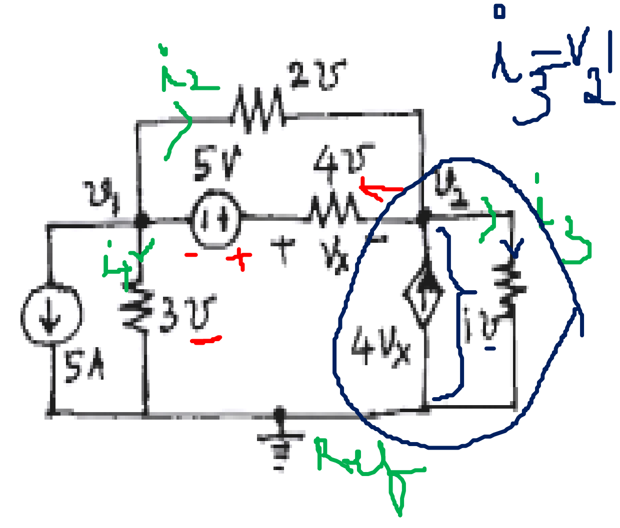
$-\left(\frac{1}{3} + \frac{1}{2}\right)V_1 + \left(\frac{1}{3} + \frac{1}{2}\right)V_2 - \left(\frac{1}{2} + \frac{1}{4}\right)V_3 = 0 \rightarrow (3)$

$V_1 = 0.667V$

$V_2 = 3.067V$

$V_3 = 2.67V$

3. Setup nodal equations for the circuit of fig and then find the power supplied by 5V source.



$V_x = 5 + V_1 - V_2$

$V_2 + V_x - 5 - V_1 = 0$

$-V_1 + V_2 + V_x = 5 \longrightarrow (1)$

$-5 - i_1 - \cancel{i_2} + \cancel{i_2} + 4V_x - i_3 = 0$

$-5 - (V_1 - 0)3 + 4V_x - (V_2 - 0)1 = 0$

$-3V_1 - V_2 + 4V_x = 5 \longrightarrow (2)$

$-3V_1 - V_2 + 4(5 + V_1 - V_2) = 5$

$-3V_1 - V_2 + 20 + 4V_1 - 4V_2 = 5$

$V_1 - 5V_2 = -15 \longrightarrow (3)$

$I = \frac{V}{R}$; $\frac{1}{R} = G$

$I = VG$

Conductance

Duality In Electric Circuits

Principle of Duality:

- Principle of duality in context of electrical networks states that “A dual of a relationship is one in which current and voltage are interchangeable”
- Two networks are dual to each other if one has mesh equation numerically identical to others node equation

List of Dual Pairs:

- For evaluating a dual network, you should follow these points
- The number of meshes in a network is equal to number of nodes in its dual network
- The impedance of a branch common to two meshes must be equal to admittance between two nodes in the dual network
- Voltage source common to both loops must be replaced by a current source between two nodes
- Open switch in a network is replaced by a closed switch in its dual network or vice versa

SI No	Elements	Dual Elements
1	Voltage (v) ; v = iR	Current (i) ; i = vG
2	Short Circuit	Open Circuit
3	Series	Parallel
4	Norton	Thevenin
5	Resistance (R)	Conductance (G)
6	Impedance	Admittance
7	KVL	KCL
8	Capacitance	Inductance

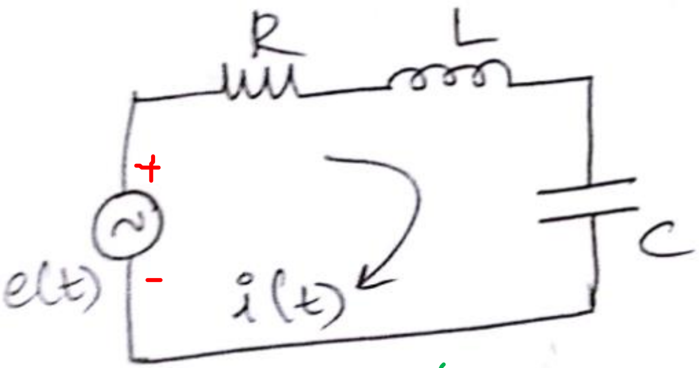
Construction of Dual Networks

Capacitors	Inductors
$i = C\dfrac{dv}{dt}$	$v = L\dfrac{di}{dt}$
$v_C(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i_C(\tau)\,d\tau$	$i_C(t) = \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v_L(\tau)\,d\tau$

$i_L(t)$

1. Mathematical Method

1. Draw dual network of a given network. Use mathematical method.



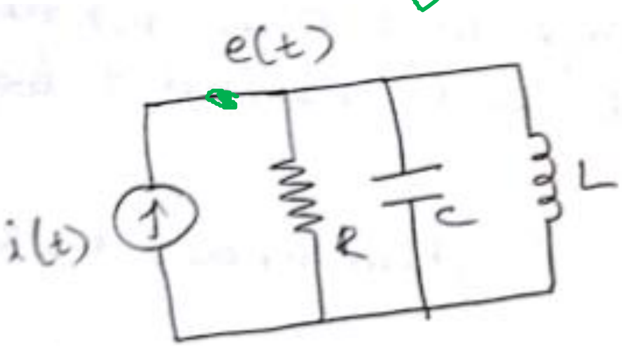
$$e(t) - R[i(t)] - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0$$

Apply KVL to the

$$e(t) - R\,i(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0 \longrightarrow (1)$$

dual

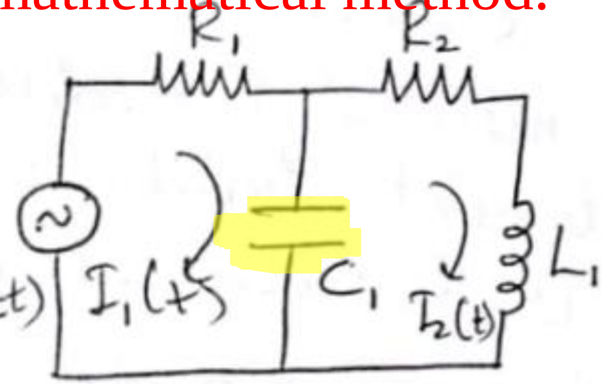
$$i(t) - G\,e(t) - C\frac{d\,e(t)}{dt} - \frac{1}{L}\int e(t)\,dt = 0$$



Writing dual elements for each & every element in eqⁿ (1),

$$i(t) - G\,e(t) - C\frac{d\,e(t)}{dt} - \frac{1}{L}\int e(t)\,dt = 0. \longrightarrow (2).$$

2. Draw dual network of a given network. Use mathematical method.

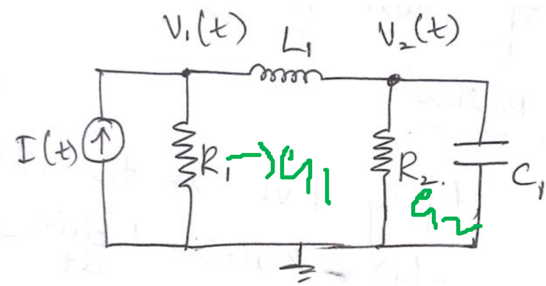


Apply KVL to loop ①.

$$E(t) - R_1 I_1(t) + \frac{1}{C_1}\int [I_2(t) - I_1(t)]\,dt = 0.$$

$$-R_1 I_1(t) + \frac{1}{C_1}\int [I_2(t) - I_1(t)]\,dt = -E(t).$$

$$R_1 I_1(t) - \frac{1}{C_1}\int [I_2(t) - I_1(t)]\,dt = E(t) \longrightarrow (1)$$



Apply KVL to loop ②

$$-R_2 I_2(t) - L_1\frac{dI_2(t)}{dt} + \frac{1}{C_1}\int [I_1(t) - I_2(t)]\,dt = 0.$$

$$R_2 I_2(t) + L_1\frac{dI_2(t)}{dt} - \frac{1}{C_1}\int [I_1(t) - I_2(t)]\,dt = 0 \longrightarrow (2).$$

Replacing each & every element in eqn (1) & (2) by its dual element we have,

$$G_1 V_1(t) - \frac{1}{L_1}\int [V_2(t) - V_1(t)]\,dt = I(t) \longrightarrow (3).$$

$$G_2 V_2(t) + C_1\frac{dV_2(t)}{dt} - \frac{1}{L_1}\int [V_1(t) - V_2(t)]\,dt = 0 \longrightarrow (4)$$

2. Graphical Method

Procedure for drawing dual network

i) First identify independent loops in a given network.

ii) Place a non-zero node inside each independent loop, name them.

iii) Place a zero potential node, (datum node) outside a given network.

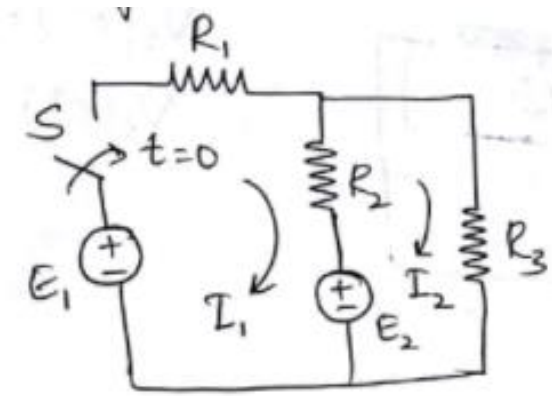
iv) If the element is exclusively in a mesh (1), then that element should be connected between datum node & node (1). Draw a dotted line from datum node to the node inside mesh through the element.

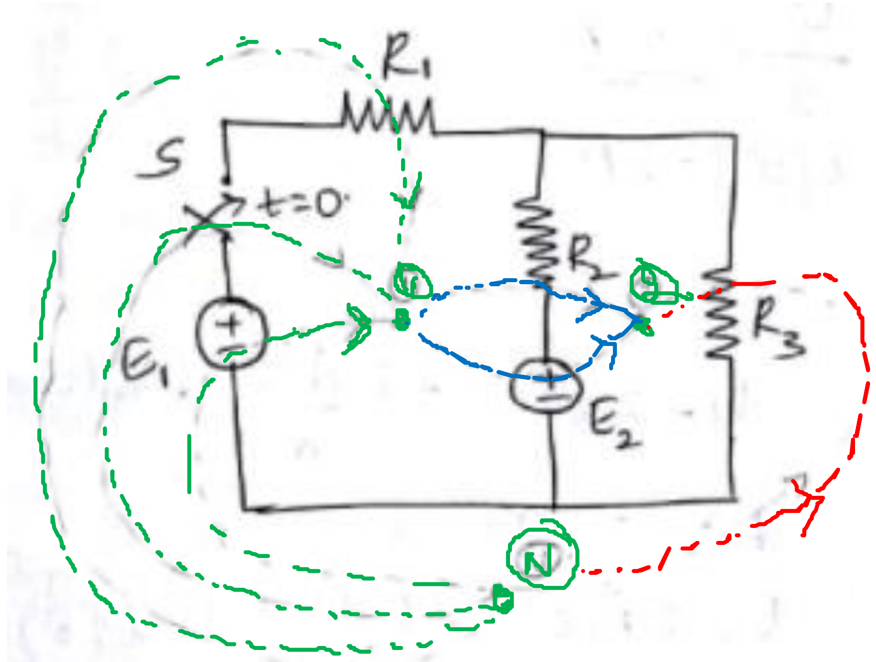
v) If the element is common between mesh (1) & mesh (2) then in dual network that dual element should be connected between two nodes node (1) & node (2).

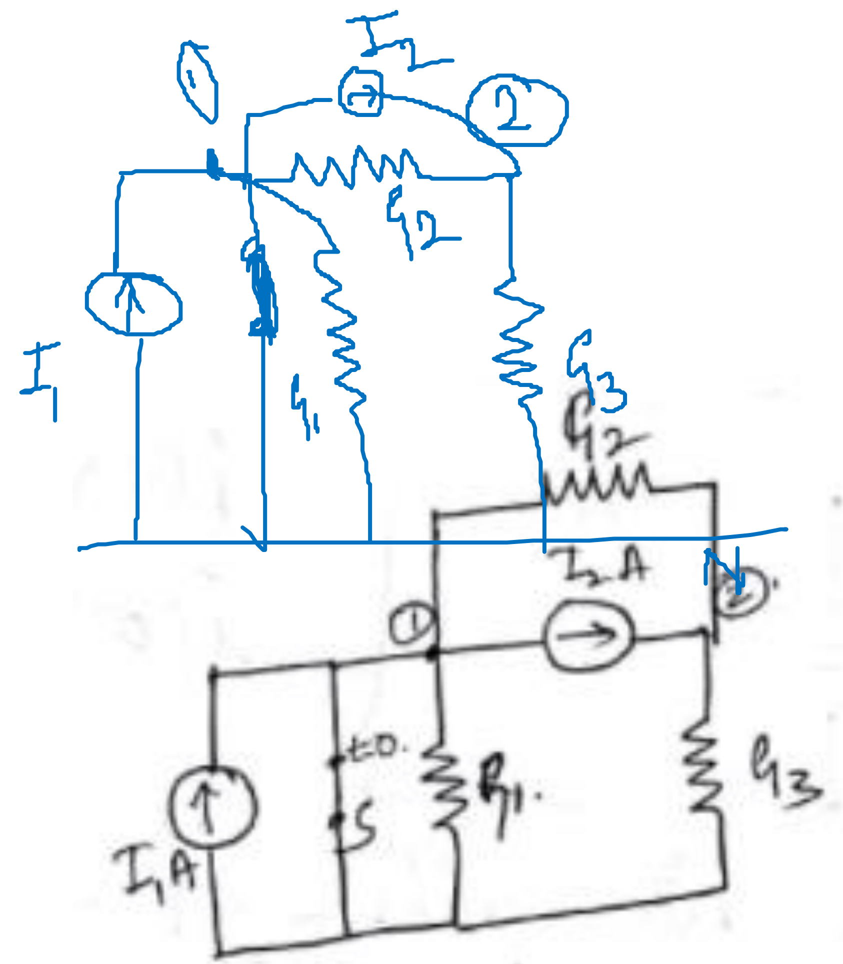
vi) Consider branch containing active source as a separate branch.

vii) While travelling in a closed path i.e., mesh in direction of loop current, if it is voltage rise then in the network the current source should be shown such that current flows towards node. If there is a voltage drop then current should be flowing away from node in dual network.

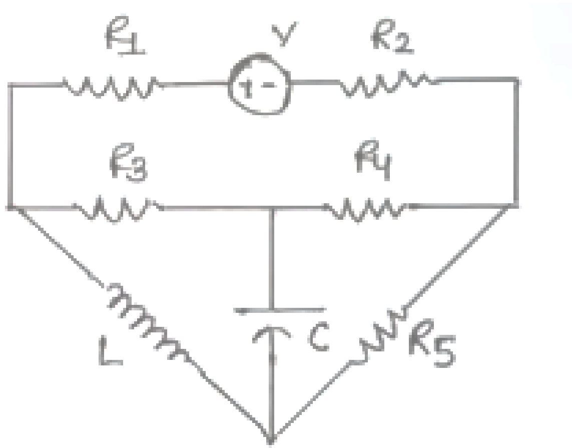
3. Draw dual network of a given network.
Use graphical method.

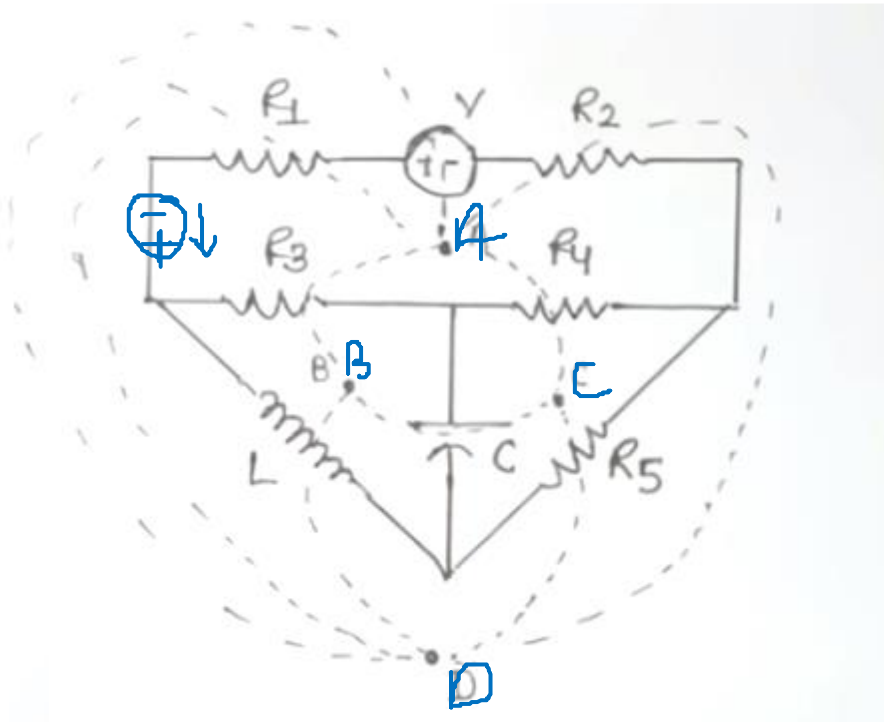


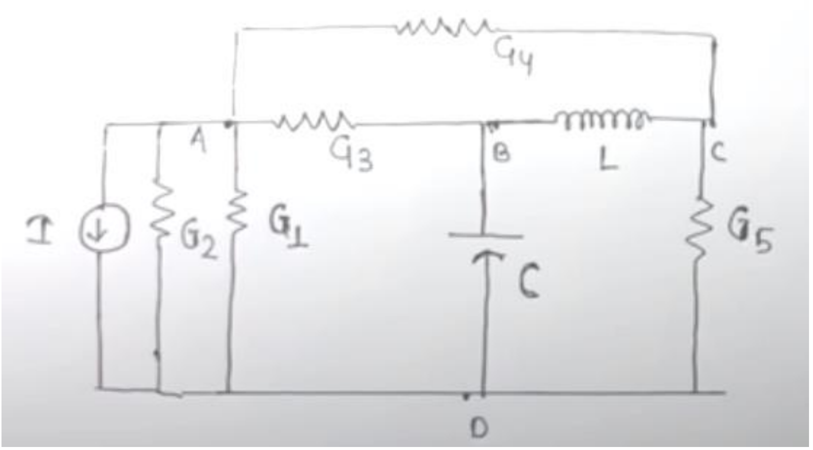




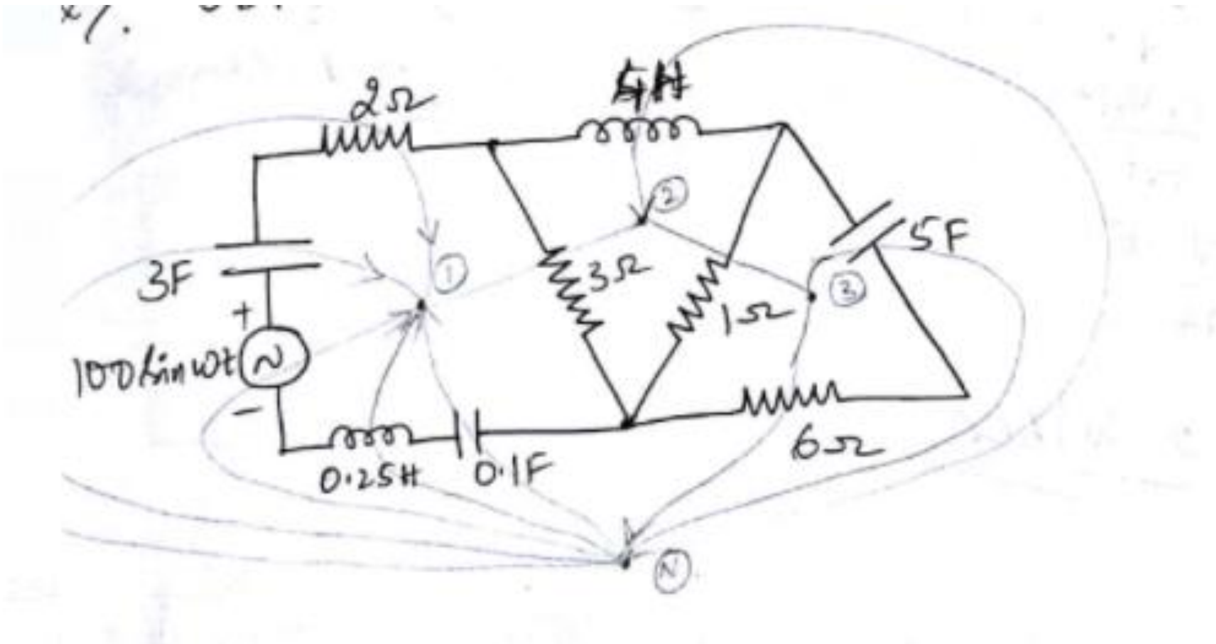
4. Draw the dual of the network in figure.







5. Draw dual network of a given network. Use graphical method.



6. Draw the exact dual of the network shown in fig by writing Kirchhoff's law equations.